

On geometric potentials in quantum-electromechanical circuits

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Abstract. In this paper, we demonstrate the formation of confinement potentials in suspended nanostructures induced by the geometry of the devices. We then propose a set-up to measure the resulting geometric-phase change in electronic wave functions in these mechanical nanostructures. The device consists of a suspended loop through which a phase-coherent current is driven. A combination of two and more geometrically induced potentials can be applied for creating mechanical-quantum-bit states.

Quantum mechanics in curved-linear manifolds has been elaborated previously [1]. The propagation of waves in curved waveguides can be ‘translated’ for quantum particles into a Hamiltonian consisting of the kinetic energy operator and a resulting potential energy, which is of pure geometric origin. The ability to build nanostructures with a three-dimensional (3D) relief allows the realization of low-dimensional electronic systems possessing a mechanical degree of freedom [2]. This is exemplified in recent works by Prinz *et al* [3] and Schmidt and Eberl [4], who demonstrated how to realize rolled-up semiconductor films with a radius of curvature $R \sim 100$ nm. Thus it is worthwhile studying the influence of geometric potentials on phase-coherently propagating particles in curved low-dimensional electron systems. This will induce a phase shift in the electronic wave function corresponding to Berry’s phase [5].

In the case of a 2D electron gas, flexing the gas leads to a geometric potential of the form

$$U = -\frac{\hbar^2}{8m} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)^2, \quad (1)$$

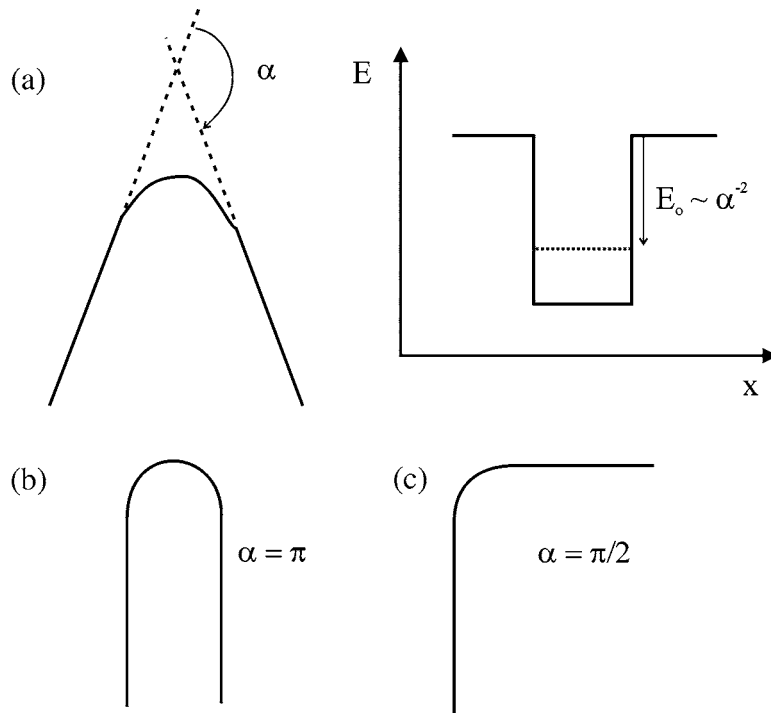


Figure 1. (a) Sketch of a single wire and the resulting geometrically induced confinement potentials. The parameter α represents the degree of bending and E_0 represents a bound electronic state in the resulting potential. This geometric potential can be represented as a rectangular potential well having a width αR and a depth $-\hbar^2/8mR^2$. (b) For $\alpha = \pi$ and $\pi/2$, square well potentials with different binding energies are obtained.

where m is the effective mass and R_1 , R_2 are the principal curvature radii of the surface at the point where the electron resides. The geometric potential is always attractive and is independent of the electric charge of a particle, similar to gravitation. Furthermore, it is of purely quantum origin, i.e. it vanishes for the limit $\hbar \rightarrow 0$.

If one of the radii tends to infinity, we obtain a cylindrical surface. Particularly, this is the case when electrons are confined to a quantum wire having the shape of a plane curve. The Schrödinger equation for such a curved-linear 1D system reads (see figure 1(a))

$$\hat{H}\psi = -\left[\frac{\hbar^2}{2m} \frac{d^2}{ds^2} + \frac{\hbar^2}{8mR^2(s)}\right]\psi = E\psi, \quad (2)$$

where s is the length of the arc of the wire counted from an arbitrary origin and $1/R(s)$ is the local curvature. A straightforward derivation of equation (2) is given in [6].³ We have further

³ In this specific case, the kinetic energy operator has the conventional form of a 1D Schrödinger equation, obtained by replacing 'x' with 's'. A curve does not possess an internal geometry, in contrast with the 2D case and manifolds of higher dimension.

shown in this paper that a wire having the shape of an Archimedes spiral gives the geometric potential with an asymptotic behaviour resulting in Coulomb's law

$$1/R^2(s) \sim 1/s. \quad (3)$$

Hence, there is an infinite set of bound states corresponding to the localization of electrons at the origin of the spiral.

If one starts with a quantum wire consisting of two straight lines conjugated by an arc of a circumference ('open book' shape), the corresponding geometric potential is a rectangular potential well (see figure 1(b)) with a width αR and a depth $-\hbar^2/8mR^2$, where R is the radius of the circumference and α is the conjugating arc (the angle between two rectilinear parts of the wire). There exists one and only one bound state for $\alpha < \pi$ in such a system and its energy is given by

$$E_0 = -\frac{\hbar^2}{8mR^2} \left(1 - \frac{16z^2}{\alpha^2}\right), \quad (4)$$

where $z(\alpha)$ is the root of $\cos z = 4z/\alpha$ between 0 and $\pi/2$ (see figure 1(b)). For example, for a U-shaped wire ($\alpha = \pi$) with $R = 100 \text{ \AA}$ and an electron mass $m = 0.07m_0$ in GaAs, we find a binding energy of $E_0 = 4 \text{ K}$, whereas for the conjugation of two perpendicular straight lines, i.e. $\alpha = \pi/2$, we obtain $E_0 = 3 \text{ K}$. The phase of the wave function in the quasiclassical regime is then given by the integral

$$\frac{1}{\hbar} \int_0^S P ds' = \frac{1}{\hbar} \int_0^S \sqrt{2m(E_F - U(s'))} ds'. \quad (5)$$

For a wire with a small curvature, the relation for the Fermi energy and the geometric potential is $E_F \gg \hbar^2/8mR^2$. Then, the total phase shift, $\Delta\phi$, after passing through the conjugation, is given as $\Delta\phi \cong \alpha/8k_F R$, where $k_F R \gg 1$ with $k_F = P_F/\hbar$. In the opposite limit $k_F R \ll 1$, the integral in equation (5) gives $\Delta\phi = \alpha/2$.

The most realistic case, of course, is given by the limit $k_F R \gg 1$. A mechanically deformable quantum interferometer will be able to sense such a deformation if its sensitivity to the phase shifts exceeds the value $\alpha/8k_F R$. Such a mechanical quantum interferometer (MQUI) can be realized by suspending a 2D electron gas in a thin membrane. While the electron gas usually is 10 nm thin, the total membrane thickness will be about 90 nm. This leaves the minimal radius of curvature at $R \sim 500 \text{ nm}$, giving a confinement potential for the lowest state of $E_0 \sim 1.6 \text{ mK}$. Prinz *et al* [3] have shown that smaller curvature radii can be achieved. In particular, considering surface-bound 2D electron gases in InAs, heterostructures should allow reaching the regime $R \sim 10 \text{ nm}$, effectively leading to temperatures in the range of $\sim 4 \text{ K}$.

An MQUI is shown in figure 2: the interferometer basically consists of a ring-shaped suspended membrane containing a 2D electron gas. This geometry first allows to measure the interference induced by a magnetic field applied perpendicular to the plane of the ring, i.e. 'classical' Aharonov–Bohm oscillations (see figure 2(a)). The interferometer has to be calibrated in the following manner: the diameter of the ring should be smaller compared with the phase coherence length L_ϕ ; hence, in this case, a diameter of $5 \mu\text{m}$ as found for 2D electron gases will be sufficient. To facilitate deformation of the arms of the ring, gating electrodes are used beneath the ring, as shown in figure 2(b). To prevent depletion of the electron

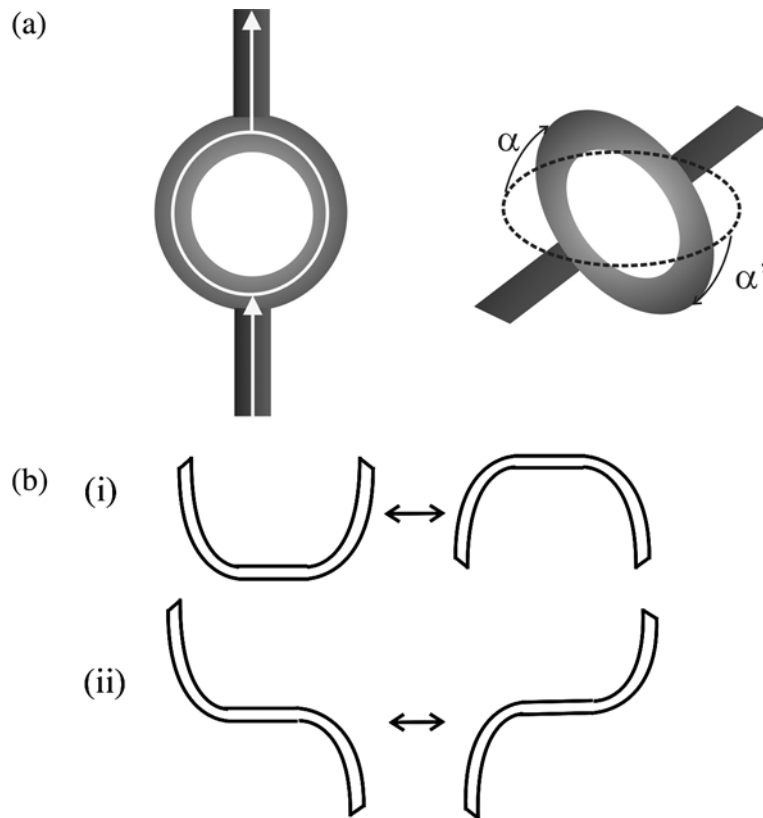


Figure 2. Mechanical quantum interferometer: (a) electrons propagate phase-coherently through the ring similar to an Aharonov–Bohm geometry. By flexing the arms of the ring (α , α'), a geometric potential is formed, which leads to an effective phase shift of the electronic wave function within the interferometer. (b) Two flexing modes can be distinguished, (i) and (ii), both of which lead to an identical phase shift.

gas, the suspended heterostructure contains a highly n-doped GaAs back-layer. By gating the arms of the interferometer individually, two flexing modes can be supported. Both modes are shown in figure 2(b). Since the potential shows a dependence of $\sim R^{-2}$, both modes will lead to the same wave function shift. Another technique for mechanically modulating the suspended electron ring involves the use of surface acoustic waves [7]. The modulation amplitude is considerably increased in suspended membranes [8].

Intriguingly, the combination of two curved sections connected by a thin wire w —denoted as a Π -shaped element—will lead to a double-quantum well potential, as demonstrated in figure 3(a). Depending on $R(s)$ and w , the two discrete states in the wells $E_0^{A,B}$ can communicate, i.e. a tunnel splitting of the order of $2\delta E_0$ will occur. Thus this system represents a mechanical quantum bit (mqubit), whose communication is given by the exchange of phonons at an energy $2\delta E_0 = hf_{\text{ph}} = hc_a/w$, where c_a is the velocity of sound in the heterostructure and w the length of the connecting element. Naturally, this scheme can be extended to a chain with N elements forming $N/2$ Π -wire mqubits, as shown in figure 3(b). A variety of different modes is available for exchange of information between two mqubits (figure 3(b), (ii)). Stretching and contracting the wire elements individually allows us to steer inter- and intra-mqubit communications. The

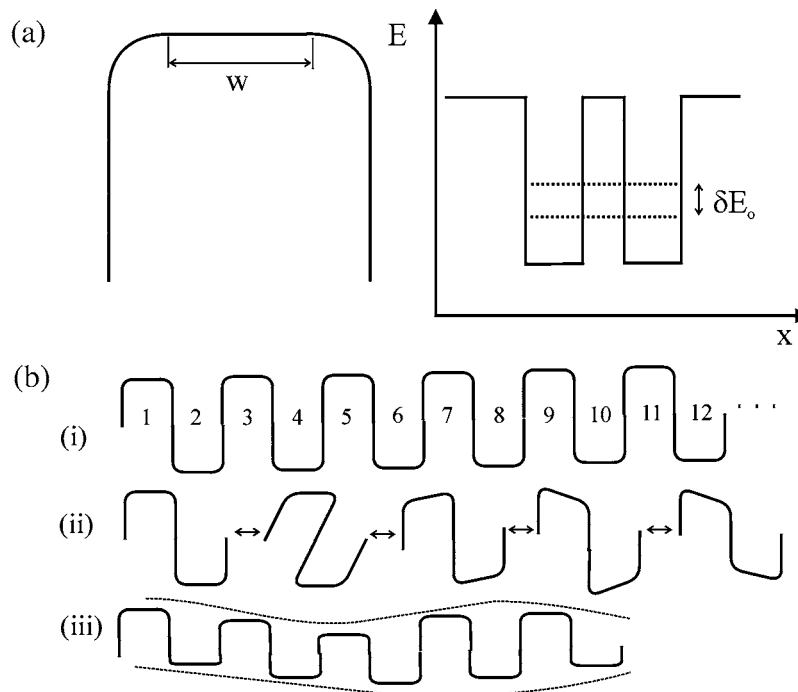


Figure 3. (a) A Π -wire forming a double-quantum well potential: the discrete states in the two wells can interact depending on the length of the connecting wire element, w . In the case of tunnelling, a mqubit is formed, i.e. the two discrete states are tunnel-split by δE_0 . (b) Chain of Π -wire elements defining a circuit of 10 coupled mqubits (i). Communication between two mqubits is achieved by a variety of local deformations of the wires (ii). Parallel addressing of mqubit chains is possible through acoustic phonons propagating along the wire with the velocity of sound (iii), but at lower frequencies (indicated by dashed lines).

overall information exchange is performed through low-frequency phonon modes (indicated by dashed lines in (iii)). The energy of these modes is determined from c_a and the chain length. By sectioning into subchains, the interaction of the mqubits is organized in a hierarchy.

As another example, we would illuminate the U-shaped wires for generating electron-hole pairs that form excitons. The electrons, holes and, hence, the excitons will be attracted to the bottom of the U-wires' geometric potentials. Since excitons do not obey the Pauli-exclusion principle, the probability of capturing more and more excitons is high.

Acknowledgments

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