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Statistics of conductance oscillations of a quantum dot in the Coulomb-blockade regime

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Abstract. – The fluctuations and the distribution of the conductance peak spacings of a quantum dot in the Coulomb-blockade regime are studied and compared with the predictions of random matrix theory (RMT). The experimental data were obtained in transport measurements performed on a semiconductor quantum dot fabricated in a GaAs-AlGaAs heterostructure. It is found that the fluctuations in the peak spacings are considerably larger than the mean level spacing in the quantum dot. The distribution of the spacings appears to be Gaussian both for zero and for non-zero magnetic field and deviates strongly from the RMT predictions.

Advanced nanofabrication techniques have made it possible to confine small numbers of electrons electrostatically within the two-dimensional electron gas (2DEG) of a semiconductor heterostructure [1], [2]. Both the electric charge and energy of such "quantum dots" are quantised and hence such structures are sometimes referred to as "artificial atoms" [3], [4]. In transport measurements the charging of these electron islands with single electrons leads to the observation of periodic conductance oscillations in the Coulomb-blockade regime [1]. These reflect the electrostatic coupling of the quantum dot to its environment and, additionally, they contain information about the eigenenergies and eigenfunctions of the electrons in the dot. Due to irregularities in the electrostatic confinement potential and electron-electron interactions, the corresponding classical motion of the electrons in the quantum dot can be expected to be chaotic (nonintegrable) [5]-[7]. Consequently, recent experiments have considered the peak height distribution [8], [9], parametric conductance correlations [9] and level statistics [10] of a quantum dot in the Coulomb-blockade regime to test the concepts developed for the quantum-mechanical description of classically chaotic systems ("quantum chaos" [11], [12]). In particular, random matrix theory (RMT) [13] has proven to be a very successful description of the statistical properties of spectra of many irregular systems. Therefore, it is a very interesting

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question, how RMT applies to the transport properties of quantum dots. In this paper we investigate the fluctuations of the peak spacings of the conductance peaks of a quantum dot obtained in transport measurements with and without applied magnetic field. The spacing distributions are calculated and compared with the predictions of RMT.

For sufficiently low temperatures T and small dot capacitances C, quantum dots isolated from the reservoirs of the 2DEG via tunnel barriers can exhibit Coulomb-blockade phenomena. When $e^2/C \gg k_{\mathrm{B}}T$, transport through the quantum dot is blocked. A finite conductance only occurs when the total energy of the quantum dot with N electrons is degenerate with the energy of the dot occupied by $N+1$ electrons. This is the case when

$$F(N+1) - F(N) = \mu \,, \tag{1}$$

where $F(N)$ denotes the free energy of the quantum dot with N electrons and μ is the chemical potential of the leads. Then a single electron can tunnel from a reservoir into the dot [1]. This can be achieved by tuning the dot's potential with a centre gate. A sweep in the centre gate voltage V_{g} results in the well-known conductance oscillations in the Coulomb-blockade regime. From eq. (1) the difference ΔV_{g} between gate voltages at which two adjacent peaks occur can be related to the thermodynamic quantity $\partial\mu/\partial N$, which has the meaning of an inverse compressibility [10]. Within the capacitive charging model [1] the electrons are assumed to occupy single-particle states of energies ϵ_i and the Coulomb interactions are described by a classical electrostatic term $U(N)$. The dot's energy is then $F(N) \approx \sum_i^N \epsilon_i + U(N)$ and the difference ΔV_{g} is given by

$$e\alpha\Delta V_{\mathrm{g}}^N = e^2/C + \Delta\epsilon_N \,. \tag{2}$$

Here e denotes the electronic charge, C the total capacitance of the dot and $\Delta\epsilon_N = \epsilon_{N+1} - \epsilon_N$ the level spacing. The conversion factor $\alpha = C_{\mathrm{g}}/C$, where C_{g} is the dot-to-gate capacitance, translates between the energy and the voltage scale of the conductance oscillations. Thus, in principle, one should be able to extract the energy level spacings $\Delta\epsilon_N$ from the so-called "addition spectrum" obtained in Coulomb-blockade measurements.

From the addition spectrum, one can calculate the nearest-neighbour spacing (NNS) distribution $P(S)$, which can be compared to the predictions of RMT. $P(S)$ is the distribution of the spacings between adjacent levels of an energy spectrum, where the spacings S are normalised to a mean value of unity. The results for $P(S)$ within the framework of RMT are very well approximated by the Wigner surmise, which is [12]

$$P(S) = \frac{\pi}{2} S e^{-\frac{\pi}{4}S^2} \qquad \text{(GOE)}\,, \tag{3}$$

$$P(S) = \frac{32}{\pi^2} S^2 e^{-\frac{4}{\pi}S^2} \qquad \text{(GUE)}\,, \tag{4}$$

for time-reversal invariant systems and for systems with broken time-reversal invariance, *e.g.*, in the presence of a magnetic field. The first distribution corresponds to the energy spectrum of Hamiltonians drawn from the Gaussian orthogonal ensemble (GOE) of random matrices, while the second is obtained for the Gaussian unitary ensemble (GUE). The fluctuations $\delta S = (\langle S^2\rangle - \langle S\rangle^2)^{1/2}$ are thus expected to be $0.52\langle S\rangle$ and $0.42\langle S\rangle$ for GOE and GUE, respectively.

The quantum dot on which our measurements were performed was defined by electron-beam lithography in the 2DEG of a GaAs-$Al_{0.32}Ga_{0.68}As$ heterostructure. The mobility and the sheet density of the 2DEG are 120 m^2/Vs and 3.6×10^{15} m^{-2}, respectively. The application of negative gate voltages to the surface structure defines an island which is isolated from the left and right reservoirs via tunnel barriers (see inset of fig. 1). The radius of the island

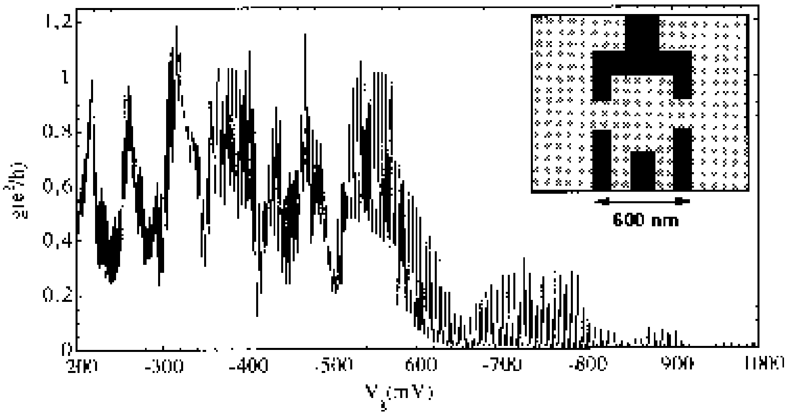

Fig. 1. – Conductance oscillations of a quantum dot in the Coulomb-blockade regime at zero magnetic field as a function of the centre-gate voltage V_g. Roughly 170 peaks are observed between $V_g = -200$ mV and $V_g = -1000$ mV. The inset shows a schematic of the quantum dot. The shaded area denotes the 2DEG and the black areas indicate the gates with which the dot is defined. The lower middle gate is the centre-gate.

is estimated to be $r \leq 150$ nm. The Coulomb-blockade measurements were performed in a dilution refrigerator with a base temperature of 25 mK. Electron transport through the dot was studied by applying a small bias voltage (4.3 μV AC) between the left and right reservoirs and measuring the current using standard lock-in techniques (for further details see ref. [14]). From the onset of the conductance oscillations at $V_g = -200$ mV roughly 170 peaks are observed (fig. 1). During the gate sweep from $V_g = -200$ mV to -1000 mV the number of electrons in the quantum dot thus varies from $N \approx 250$ to $N \approx 80$. In the following, the only relevant energy scale is the mean energy level spacing $\varDelta$. It should be roughly E_F/N, where E_F is the Fermi energy. From the sheet density one obtains $E_F \approx 12.9$ meV and therefore $\varDelta \approx 50$ μeV. The thermal energy $k_B T$ is about one order of magnitude smaller.

To calculate the NNS distribution from the conductance oscillations, first the gate voltage differences ΔV_g between adjacent peaks are extracted from the data. The mean value of ΔV_g increases linearly with decreasing voltage (see fig. 2 *a*)) reflecting an inverse linear change in the dot-to-gate capacitance [15]. Identifying $\langle \Delta V_g \rangle$ with the classical charging voltage e/C_g, from eq. (2) the energy spacings are obtained as

$$\Delta\epsilon = e\alpha(\Delta V_g - \langle \Delta V_g \rangle)\,. \tag{5}$$

The conversion factor α is also a function of the gate voltage. This can be considered by using the same linear fit as above, *i.e.* $\alpha = C_g/C = e(e + C_{\text{rest}} \cdot \langle \Delta V_g \rangle)^{-1}$, where the capacitance $C_{\text{rest}} = C - C_g$ is assumed to be constant. However, the actual choice of α is not a crucial parameter in the calculation, as tests with different constant values for α have shown. From eq. (5) the $\Delta\epsilon$ are obtained as fluctuations around a mean value of zero. To remove the unphysical negative values for $\Delta\epsilon$ the whole data are shifted by a constant value (cf. fig. 2 *b*)). It turns out that the fluctuations around the mean value are considerably larger than the mean level spacing $\varDelta$ estimated above. This may be regarded as an indication that the calculated $\Delta\epsilon$ are not the "real" addition energies and that the influence of the electron-electron interactions both within the dot and its environment play a significant role [10].

From the energy spacings one can construct an artificial one-particle energy spectrum via $E_i = \sum_{N=1}^{i} \Delta\epsilon_N$. To unfold the data to a mean level spacing of unity, a polynomial fit is made

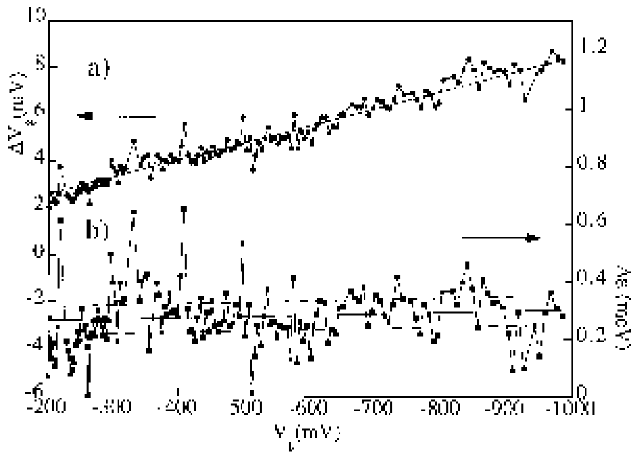

Fig. 2. – *a*) The peak spacings (ΔV_g) extracted from fig. 1 (indicated by dots) and a linear fit to them. *b*) Energy spacings ($\Delta\epsilon$) calculated from the peak spacings and shifted to positive values. The straight line indicates their mean value and the broken lines indicate the largeness of the fluctuations expected from RMT.

to the spectral step function $N(E) = \sum_i \theta(E - E_i)$. The renormalized energies are obtained by the standard unfolding mapping $E_i \mapsto \langle N(E_i) \rangle$ [11], [12]. From these, the energy level spacings are calculated and can be directly compared to the predictions of RMT.

In fig. 3 *a*) the resulting NNS distribution for zero magnetic field is displayed. It obviously does not agree with the Wigner surmise (eq. (3)). Instead, it is much better described by a Gaussian centered at $S = 1$, as illustrated.

In the presence of a magnetic field, time-reversal invariance breaks down. In this case the appropriate ensemble of random matrix theory to describe energy level fluctuations is the Gaussian unitary ensemble. However, as in the $B = 0\,\mathrm{T}$ case, the experimentally obtained spacing distributions look Gaussian rather than GUE-like. In fig. 3 *b*) the distributions for zero, for low ($B = 0.1\,\mathrm{T}, B = 0.5\,\mathrm{T}$) and high ($B = 4\,\mathrm{T}$) magnetic fields are displayed. The distributions are derived with typically 150–170 data points. From a statistical point of view this number is rather small. Nonetheless, these are large numbers when compared to previous Coulomb-blockade experiments [10]. It can be seen that the distributions narrow with increasing magnetic field as may be expected due to the Landau quantisation [16], [17].

The largeness of the fluctuations indicates that the capacitive term e^2/C in eq. (2) undergoes even larger fluctuations than the energy levels themselves. Thus, the $\Delta\epsilon$ obtained above do not display the energy level spectrum of the quantum dot. However, this would not mean a failure of RMT, but an insufficiency of the capacitive charging model. Equation (2) obviously cannot be used to get access to the bare energy level spacings of the quantum dot, when a larger range of gate voltages is considered.

In a recent publication Sivan *et al.* [10] argued that electron-electron interactions in the dot were responsible for the failure of RMT to describe the conductance peak spacing distribution. Their experiments and calculations lead to a Gaussian $P(S)$ centered at $S = 1$ which is similar to our results. In terms of the charging energy the fluctuations obtained in our experiment are $\delta(\Delta\epsilon) \approx 0.07\text{–}40.11 e^2/C$, which is slightly smaller than in the work by Sivan *et al.* Their numerical calculations suggest that fluctuations in the quantity $\Delta V / \langle \Delta V \rangle$ converge to a "universal" value between 0.1 and 0.2 for strong electronic interactions. Calculating this quantity from our data we arrive at 0.10, which is consistent with their finding. However,

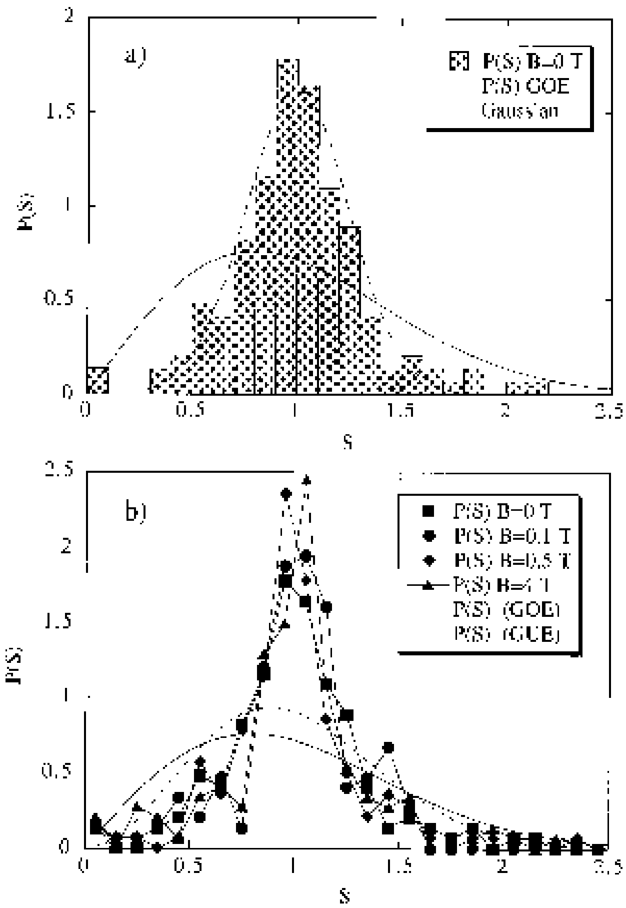

Fig. 3. – *a*) NNS histogram calculated from the energy spacings of fig. 2 *b*) after unfolding them to a mean value of unity. The full line denotes the GOE prediction of RMT for $P(S)$ and the broken line is a Gaussian fit centered at $S = 1$. *b*) NNS distributions for different magnetic field strengths. The full and the broken lines denote the RMT predictions for $P(S)$ obtained in the Gaussian orthogonal and unitary ensemble, respectively. For clarity, lines have been used to display the distributions instead of histograms.

the influence of the capacitive coupling to the reservoirs has not been considered in their publication, which may also have a considerable influence on the fluctuation properties of the peak spacings.

Finally, it has to be considered that RMT was initially developed to handle the statistical properties of excitation spectra of complex systems. The addition spectrum as obtained in Coulomb-blockade measurements, however, consists of the many-particle ground state energies of the quantum dot rather than excitation energies. The comparison with RMT has been made under the assumption that the addition spectrum be equivalent to a single particle spectrum. Indeed, the excitation spectrum of the model used in [10] obeys RMT. Recently, this could also be shown for the excitation spectrum of the two-dimensional Hubbard model [18]. But it is not clear whether the results of RMT can be applied to ground-state energy statistics. In this respect it is interesting to notice that the peak height distribution for the conductance oscillations seems to be in accordance with RMT [8], [9], whereas the parametric conductance correlations [9], [19] quantitatively do not agree with RMT.

In conclusion, we have investigated the statistics of conductance peak spacings obtained in Coulomb-blockade experiments with zero and non-zero magnetic field. In all cases the results do not agree with the predictions of random matrix theory. Instead, the nearest-neighbour spacings appear to be Gaussian distributed around their mean value. It seems to be difficult to extract the bare energy levels when using a simple capacitive charging model. Therefore, our results include fluctuations in the electrostatic coupling with the environment which are larger than the fluctuations in the quantum dot's energy level spectrum itself. Further theoretical and experimental work are required to understand this central phenomenon in mesoscopic physics.

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