Classical and quantum transport in rectangular antidot superlattices

R. Schuster
Braun Center for Submicron Research, Weizmann Institute of Science, Rehovot 76100, Israel and Sektion Physik, Ludwig-Maximilians Universität München, Geschwister-Scholl-Platz 1, 80539 München, Germany

K. Ensslin
Festkörperphysik, Eidgenössische Technische Hochschule Zürich and Paul Scherrer Institut, 8093 Zürich, Switzerland and Sektion Physik, Ludwig-Maximilians Universität München, Geschwister-Scholl-Platz 1, 80539 München, Germany

J. P. Kotthaus
Sektion Physik, Ludwig-Maximilians Universität München, Geschwister-Scholl-Platz 1, 80539 München, Germany

G. Böhm and W. Klein
Walter Schottky Institute, Technische Universität München, 85748 Garching, Germany
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Experiments on antidot superlattices with a variety of rectangular geometries are used to test basic symmetry relations. Depending on the direction of current flow with respect to lattice orientation, different classical periodic orbits are probed. The symmetry relations also persist into the quantum-mechanical regime. Our experimental results show that a quantum effect like the Shubnikov–de Haas oscillations can be modified for the current direction where the resistance is influenced by classical trajectories and periodic orbits. [S0163-1829(96)07447-4]

Antidot superlattices[1–4] represent a versatile system to study electron transport in periodic potentials. A periodic array of potential pillars exceeding the Fermi energy in height is superimposed on a two-dimensional electron gas (2DEG). Usually the elastic mean free path is much larger than the lattice period while the Fermi wavelength is typically an order of magnitude smaller than the characteristic features of the artificial superlattice. The electrons are considered to behave like billiard balls bouncing around in the antidot potential landscape. In a magnetic field applied perpendicular to the 2DEG the electrons travel around groups of antidots once the cyclotron diameter is commensurate with the lattice period. A calculation based on classical chaotic dynamics explains quantitatively the observed transport properties.[5] A quantum transport calculation of the longitudinal and Hall resistivity of square lattices also yields results in agreement with experimental data.[6] Experimentally pronounced maxima occur in the magnetoresistance which are related to periodic orbits around groups of antidots. The resistivity tensor of a square lattice is isotropic and no dependence on the direction of current flow is theoretically expected or experimentally observed.

In this paper we study lattices with a rectangular symmetry where the transport properties depend strongly on the direction of current flow with respect to the lattice orientation.[7] For current flow along the long lattice period the electrons are forced between the closely spaced antidots and the magnetoresistance displays pronounced maxima at magnetic fields commensurate with the lattice period. In the perpendicular direction where the electrons predominantly flow in the channels between the rows of antidots only orbits whose size is comparable with the large lattice constant manifest themselves in the magnetoresistance. In order to investigate how quantum properties arise in antidot superlattices whose classical behavior is well understood we perform measurements at low temperatures, $T<100$ mK, where both elastic and inelastic scattering lengths are larger than the length of the relevant periodic orbits. In this case additional quantum oscillations in the magnetoresistance are superimposed on the classical peaks whose periodicity depends on the direction of current flow. For the direction where the resistance is dominated by the influence of periodic orbits around single antidots we find B-periodic quantum oscillations reminiscent of Aharanov-Bohm oscillations while for the perpendicular direction $1/B$-periodic Shubnikov–de Haas (SdH) oscillations are observed. These results show that there is a close relation between quantum corrections to transport properties and classical periodic orbits. We compare our experimental results to recent semiclassical theories[8,9] and to a quantum-mechanical approach.[10] The latter theory by Neudert, Rotter, Rössler and Suhrke (NRRS) shows that the anisotropic miniband structure in a two-dimensional rectangular potential is responsible for the different quantum oscillations.

The fabrication process starts from a GaAs/$Al_xGa_{1-x}As$ heterostructure that contains a 2DEG 65 nm below the surface. Its electron density is $n_s=3\times10^{15}$ m^{-2} and the elastic mean free path is $l_e=8$ μm. Two Hall geometries which are oriented perpendicular to each other are defined by wet etching and provided with Ohmic contacts (AuGe/Ni). This setup is suitable to experimentally determine the components of the resistivity tensor. The antidot pattern is produced by electron beam lithography and transferred onto the sample by a carefully tuned wet etching step. Figure 1 shows an image of the surface of a wet etched rectangular lattice with periods $a_x=960$ nm and $a_y=240$ nm. Each antidot is well developed

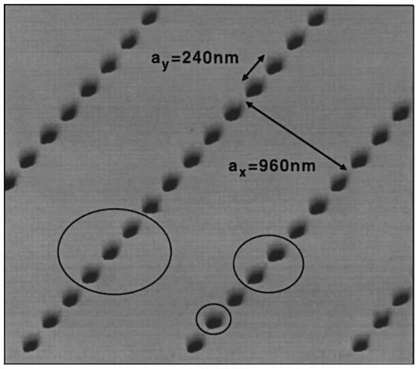

FIG. 1. Image of a wet etched surface of a GaAs/$Al_xGa_{1-x}As$ heterostructure. The rectangular antidot pattern with lattice periods $a_x=960$ nm and $a_y=240$ nm was produced by electron beam lithography. The image is taken with an atomic force microscope. If a magnetic field is applied perpendicular to the plane of the 2DEG the electrons travel around groups of antidots as it is schematically shown in the figure.

and the variation in size is remarkably small. The whole structure is covered by a gate metal which allows us to change the Fermi energy in the system. We present data from various rectangular lattices with a whole range of anisotropies between the lattice constants. In this paper we denote by a_x the long period of the lattice and by a_y the short one. For the current flow along the x direction the electrons have to flow through the closely spaced antidots and the resistivity component is ρ_{xx}. If the current flows through the wide open channels ρ_{yy} is the respective resistivity component.

Figure 2(a) presents the magnetoresistances ρ_{xx} and ρ_{yy} for a rectangular lattice with an anisotropy $a_x:a_y=480$ nm:240 nm = 2:1. A pronounced maximum occurs in ρ_{xx} at around $B\approx 1$ T. As in the case of square lattices this maximum reflects a periodic orbit around a single antidot. At smaller magnetic fields, $0\leqslant B\leqslant 0.5$ T, there is a series of further commensurability maxima corresponding to orbits around groups of antidots [see inset in Fig. 2(a)]. The orbit around two antidots which is unlikely to occur in square lattices for geometrical reasons becomes more probable in a rectangular lattice. A completely different behavior is observed when the current flows through the wide channels in the direction of the short period of the lattice. The magnetoresistance ρ_{yy} shows no structure in the regime where the pronounced maximum in ρ_{xx} occurs. The observation that commensurability oscillations mainly show up in ρ_{xx} but not in ρ_{yy} can be explained by a different coupling to the periodic orbits around the antidots.[7] The electrons traveling along chaotic trajectories within the wide channels are only slightly influenced by the presence of the periodic orbits. Only at small magnetic fields $B\leqslant 0.5$ T where the periodic orbits extend into the channels is the magnetoresistance similarly influenced for both current directions. The quantum mechanical theory by NRRS (Ref. 10) calculating the band structure for this system and the conductivity from the Kubo formula reproduces all the features of the experiment. The strongly anisotropic behavior of ρ_{xx} and ρ_{yy} can be traced to differences in the magnetic-field dependence of band and scattering contributions to the conductivity. These calculations give new insight how the dispersion of minibands corresponds to classical trajectories.

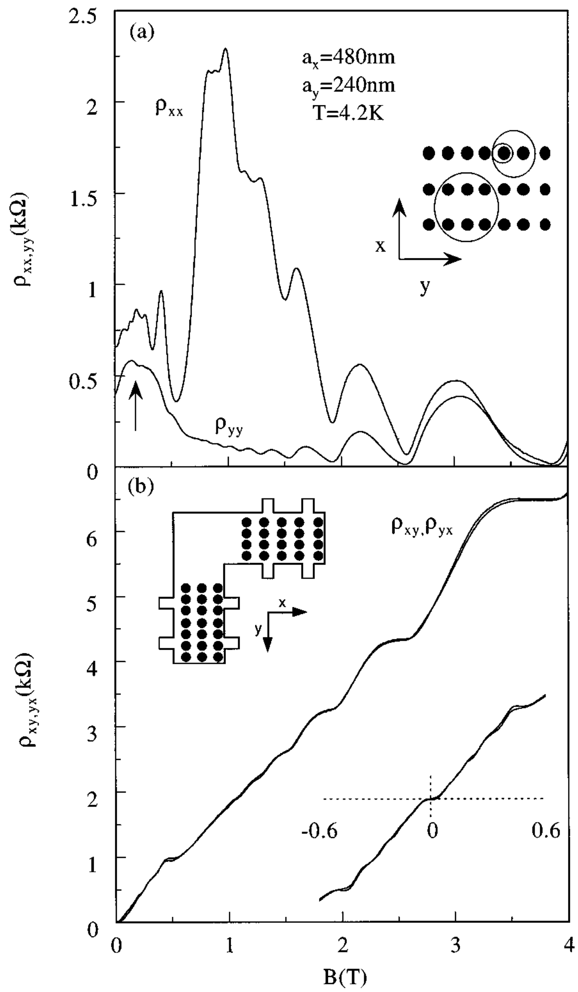

FIG. 2. (a) Magnetoresistance at $T=4.2$ K for a rectangular antidot lattice for current flow through the closely spaced antidots (ρ_{xx}) and through the wide channels (ρ_{yy}). The anisotropy between the two lateral periods is $a_x:a_y=480$ nm:240 nm=2:1. The inset in (b) clarifies the different current directions with respect to the lattice orientation. The peaks in ρ_{xx} can be ascribed to commensurate orbits around groups of antidots. An additional maximum arises at low magnetic fields for both current directions (indicated by a vertical arrow). (b) Hall resistance for the sample as measured for both current directions. The curve in the lower corner shows a magnification of ρ_{xy} and ρ_{yx} around $B=0$.

In all rectangular antidot samples a pronounced maximum is observed at small magnetic fields (Figs. 2–4). The maxima indicated by the vertical arrows in Figs. 2 and 3 occur in ρ_{xx} as well as in ρ_{yy}. In the following we argue that the maximum originates from a different physical effect, namely, the scattering of electrons in the wirelike geometry in analogy to boundary scattering in quantum wires. The magnetoresistance of quantum wires displays a maximum at low magnetic fields where the classical cyclotron radius is roughly twice as large as the width W of the wire, $0.55R_c\approx W$.[11] This effect has been first observed in thin metal films[12] and was explained by diffusive scattering at the rough boundaries of the system.[13] Diffusive scattering requires any roughness to be larger or equal to the Fermi wave-

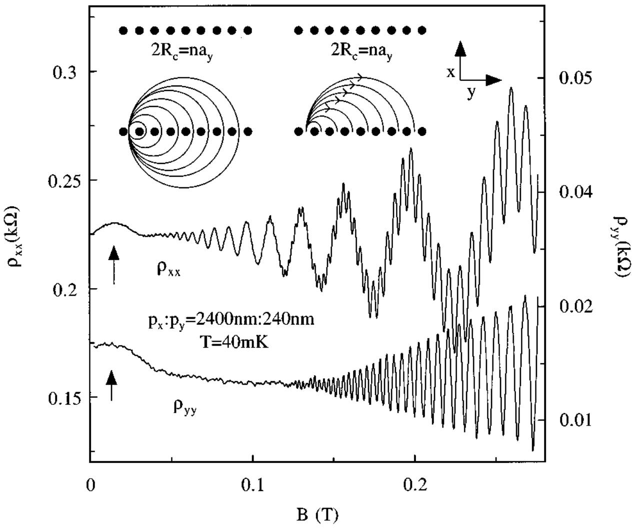

FIG. 3. Magnetoresistance for a rectangular antidot lattice for both directions of current flow. The anisotropy between the two lateral periods is $a_x:a_y=4800$ nm:240 nm=20:1. The experiment is done at $T=40$ mK. Many commensurability oscillations are observed in ρ_{xx} corresponding to electron orbits around 1,2, . . . ,18 antidots. Superimposed are SdH oscillations for $B\geqslant0.1$ T. The maximum which is related to scattering at the corrugated boundaries formed by the antidot rows is indicated by an arrow.

length λ_F. For small magnetic fields the electrons travel predominantly in the wire center and rarely reach the wire edges. At high fields skipping orbits arise along the edges of the wire. For intermediate fields the diffusion is reduced and a maximum in the resistance arises. In rectangular antidot lattices the situation is very similar. Although the scattering in our samples is mostly specular, a similar effect is likely to occur at the corrugated boundaries of the wide channels which are formed by the antidot rows. The correlation length of the roughness is now given by the short period of the lattice. The width of the quasiwire between the rows is not as well defined as in the case of single wires. An upper bound is the size of the larger period minus the diameter of the antidots. For the sample with $a_x:a_y=480$ nm:240 nm we find a value $W_{\max}\approx300$ nm compared with $W=0.55R_c\approx250$ nm from the edge roughness scattering model.

In square lattices the periodic orbits influence the magnetoresistance as well as the Hall resistance.[4] The plateaulike features in the Hall effect are related in their magnetic-field position to the occurrence of maxima in the magnetoresistance. This is explained by similar classes of trajectories that are thought to cause these two effects. The Hall effect of a rectangular lattice is presented in Fig. 2(b). While the magnetoresistance is highly anisotropic and strongly dependent on the current direction the Hall effect is isotropic within the accuracy of the experiment, $|\rho_{xy}|=|\rho_{yx}|$.[7] This is a direct consequence of Onsager's relation[14] which reflects the equality of the off-diagonal components of the resistivity tensor under reversal of the magnetic field, i.e., $\rho_{xy}(B)=\rho_{yx}(-B)$. A rectangular lattice is symmetric under reversal of the magnetic field which explains the symmetry of the Hall effect as observed in Fig. 2(b). This general symmetry relation, however, does not explain the microscopic origin of the observation.

Figure 3 presents the magnetoresistance of a lattice with an extreme anisotropy of $a_x:a_y=4800$ nm:240 nm=20:1. At $B=0$ the resistance in the barrier-dominated geometry (ρ_{xx}) is much larger than that of the wirelike geometry (ρ_{yy}) in agreement with geometrical considerations. At higher fields $B\geqslant50$ mT a remarkable number of maxima arises in ρ_{xx} while ρ_{yy} shows no classical commensurability effects. The oscillations in ρ_{xx} are exactly $1/B$ periodic. The linear behavior agrees nicely with the commensurability condition $2R_c=na_y$ in the sense that the size of the electron orbits around 1,2,3, . . . ,18 coincides with the classical cyclotron diameter (see left inset of Fig. 3). Superimposed are SdH oscillations which are also periodic in $1/B$ and which occur in both ρ_{xx} and ρ_{yy}.

To explain the effects in lattices with large anisotropies a different point of view can be adopted. If the electrons travel in the direction of the closely spaced antidots they can be backscattered once the cyclotron diameter fits a multiple integer of the short lattice period a_y (see right inset of Fig. 3). Similar electron focusing experiments have been done on specially designed parallel point contact geometries[15] where one point contact serves as an injector and a second one as a collector. If a magnetic field is applied perpendicular to the plane of the 2DEG the number of electrons which reach the collector has a maximum whenever the cyclotron diameter coincides with the distance L of the point contacts, $2R_c=L$. In antidot lattices this leads to an enhanced backscattering. The resonance condition $2R_c=na_y$ for the occurrence of a maximum is the same as if one considers periodic orbits around groups of antidots. Both points of view rely on ballistic classical trajectories in spite of their different conceptual background.

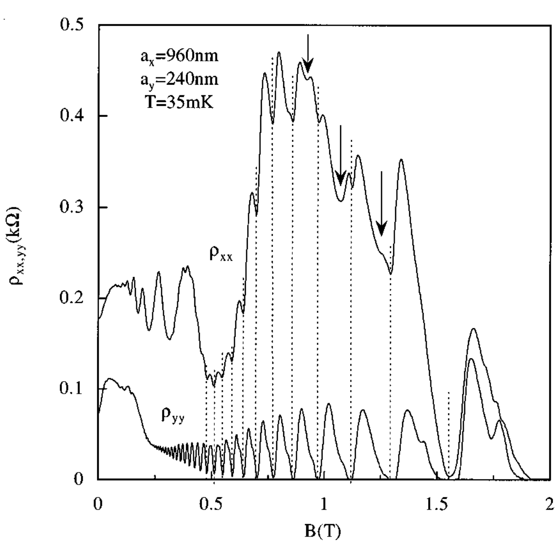

FIG. 4. Magnetoresistance for a rectangular lattice with an anisotropy of $a_x:a_y=960$ nm:240 nm=4:1. The superimposed SdH oscillations in ρ_{yy} are periodic in $1/B$ while in ρ_{xx} additional oscillations arise (see arrows).

All the data presented so far have been explained in the framework of classical dynamics. At very low temperatures, $T<100$ mK, it is possible to resolve the SdH oscillations in

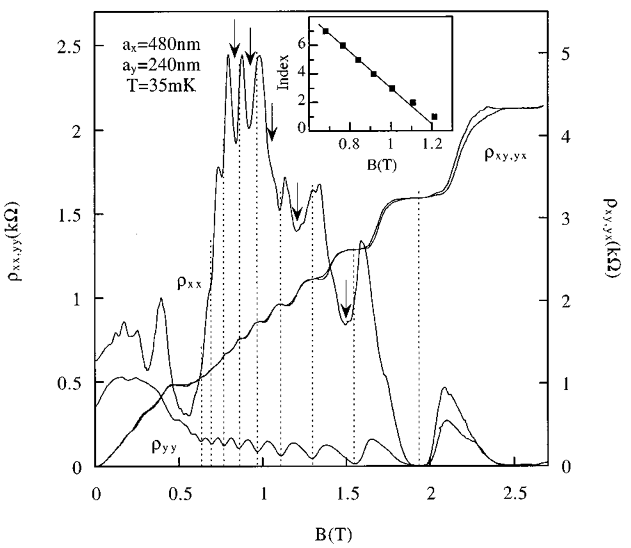

FIG. 5. Magnetoresistance and Hall resistance for a rectangular lattice with an anisotropy of $a_x:a_y=480$ nm:240 nm=2:1 at $T=30$ mK. In the magnetic-field regime, where the maximum corresponding to a classical periodic orbit around a single antidot occurs, B-periodic oscillations are observed in ρ_{xx} while the oscillations in ρ_{yy} are periodic in $1/B$. The positions of the resistance minima in ρ_{xx} which are obtained from $\Delta\rho_{xx}=\rho_{xx}(30\text{ mK})-\rho_{xx}(4.2\text{ K})$ are equidistant on the B scale (inset).

the magnetic-field range where classical commensurability oscillations occur. The data in Fig. 3 suggest that SdH (fast) and commensurability (slow) oscillations may coexist in rectangular antidot lattices with large anisotropies. Here SdH oscillations which are in phase for both current directions are exactly periodic in $1/B$ up to high filling factors $\nu\leqslant 120$. At higher temperatures, $T\geqslant 1$ K, the SdH oscillations disappear in this magnetic-field range while the classical maxima remain. There are large regions of unpatterned 2DEG's between the antidot rows so that the free cyclotron orbits are hardly influenced by the presence of the antidots.

If the antidot rows are moved closer together we expect deviations from the $1/B$-periodic behavior since the number of cyclotron orbits which are modified by the antidot potential increases. We discuss these phenomena in a different rectangular lattice presented in Fig. 4. Here the lattice periods are $a_x:a_y=960$ nm:240 nm=4:1. Again the dominant structures in ρ_{xx} are the maxima which correspond to cyclotron orbits around groups of antidots. The superimposed SdH oscillations in ρ_{yy} are periodic in $1/B$ while additional oscillations appear in ρ_{xx} (marked by arrows in Fig. 4). It is remarkable that the commensurability effects can be seen also in ρ_{yy} as an envelope of the SdH maxima. This indicates that the electrons which flow through the wide channels are also influenced slightly by the periodic orbits. If the distance between neighboring rows is reduced further the periodicity of the SdH oscillations in the two directions of current flow becomes more different. This is obvious from the data of a sample with an anisotropy $a_x:a_y=480$ nm:240 nm=2:1 which is presented Fig. 5. The oscillations in ρ_{xx}, which are superimposed on the strongly pronounced maximum at $2R_c=a_y$, deviate from the $1/B$ periodicity in ρ_{yy}. In this magnetic-field range ($0.6\leqslant B\leqslant 1$ T), where the electrons classically encircle a single antidot, the oscillations are almost B periodic (see inset in Fig. 5). For higher magnetic fields $B>1.5$ T there is a crossover to $1/B$-periodic SdH oscillations. In the quantum Hall regime the filling factors for ρ_{xx} and ρ_{yy} are identical.

As has been already shown the Hall resistance does not depend on the direction of current flow while the magnetoresistance is highly anisotropic. This general symmetry relation also applies in the magnetic-field regime where classical effects and quantum oscillations coexist. There are quantum oscillations superimposed on the classical Hall resistance (appearing as plateaulike structures in our experiment). In the magnetic-field regime $0.5\leqslant B\leqslant 1.5$ T these quantum oscillations in the Hall resistance are not in phase with the oscillations in ρ_{xx} and ρ_{yy}. Only for magnetic fields $B>1.5$ T where $a_x,a_y\geqslant 2R_c$ and commensurability effects are no longer present the plateaus in the Hall resistance take quantized values and are in phase with the SdH oscillations in ρ_{xx} and ρ_{yy}.

Weiss *et al.*[16] have observed quantum oscillations superimposed on the classical peak in square antidot lattices. Depending on the antidot potential the periodicity can change from a B periodicity similar to Aharonov-Bohm oscillations to a $1/B$ periodicity familiar from SdH oscillations. The authors have calculated the contribution of a few simple periodic orbits to the density of states using Gutzwiller's trace formula.[17] The quantum oscillations are attributed to the oscillatory structure of the density of states. Since the density of states should be isotropic this model cannot be applied to rectangular lattices with their anisotropies of the periods of the quantum oscillations. Using the Kubo formula Richter[8] and Hackenbroich and von Oppen[9] independently derived an analytical semiclassical expression for the quantum contributions to the conductivity in terms of periodic orbits. The contribution of each periodic orbit oscillates as a function of Fermi energy and magnetic field with a phase determined by its classical action $S_{\rm PO}(E_F,B)=\int \mathbf{p}\cdot d\mathbf{r}$. The amplitude depends on the stability and the velocity correlations along the periodic orbits. It is found that only a few of the infinitely many periodic orbits give a significant contribution.

This formalism provides a qualitative understanding of the relation between classical trajectories and the observed quantum oscillations. Different periodic orbits and trajectories are important for describing the transport in the two lattice directions which leads to different quantum contributions in ρ_{xx} and ρ_{yy}. We discuss the magnetic-field regime where the classical cyclotron diameter matches the short lattice period. Electron transport through the wide channels is only weakly influenced by periodic orbits. In this direction of current flow mainly unperturbed cyclotron orbits contribute to the quantum oscillations. Their action $S_{\rm PO}=eBA(B)$ leads to the well known $1/B$ periodicity of the SdH oscillations since the enclosed area $A(B)=\pi R_c^2$ scales with $1/B^2$. However, if the current flows through the closely spaced antidots the contributions of periodic orbits around single antidots play a major role. Their action $S_{\rm PO}(E_F,B)$ determines the periodicity of the quantum oscillations. Since it is difficult for the cyclotron orbits to contract with increasing magnetic field the area enclosed by the orbits deviates from the $1/B^2$ behavior. The quantum oscillations are no longer periodic in $1/B$. In the magnetic-field regime where the enclosed area remains approximately constant with magnetic field we find B-periodic oscillations. This behavior can be seen in the magnetic field range $0.6 \leqslant B \leqslant 1.1$ T (Fig. 5). The quasi-B-periodic oscillations with a period $\Delta B = h/eA \approx 90$ mT are caused by circular orbits which enclose a constant area $A = \pi(a_y/2)^2$. In the Hall resistance both the unperturbed and the modified orbits contribute to the plateaulike structures. It is difficult, however, to establish their particular influence.

The behavior of the quantum oscillations in rectangular lattices is confirmed by the theory of NRRS (Ref. 10) which evaluates the Kubo formula to obtain the components of the conductivity. Two contributions to the diagonal components can be distinguished; the band conductivity related to a nonvanishing group velocity in a dispersive miniband and the scattering conductivity associated with scattering between minibands. The influence of scattering and band conductivity in rectangular lattices is dependent on the direction and the magnetic field. For magnetic fields where $a_y < 2R_c$ the conductivity σ_{xx} is dominated by scattering contributions as in a 2DEG and SdH oscillations are found in ρ_{yy}. The other component σ_{yy} is determined by the band conductivity due to the influence of the antidot potential which gives rise to the dispersion. This leads to commensurability oscillations in ρ_{xx} superimposed by quantum oscillations out of phase with the SdH oscillations. The different periodicity of the quantum oscillations in ρ_{xx} and ρ_{yy} is a direct consequence of the magnetic-field dependence of the miniband structure in the respective lattice directions.

In summary, we have reported a series of experiments on rectangular antidot lattices that span a whole range of anisotropies of the two lattice constants. The classical periodic orbits have a different influence on the resistance traces depending on the direction of current flow with respect to the lattice. The quantum oscillations which are superimposed on the classical commensurability maxima are modified by the presence of the periodic orbits. These results contribute towards the understanding of how quantum properties may arise in superlattices whose classical dynamics is well understood.

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