Coherent detection of nonlinear nanomechanical motion using a stroboscopic downconversion technique

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A method is presented that overcomes bandwidth limitations arising in a fiber-optic setup transducing mechanical motion. The reflected light from a sample incorporating a nanomechanical resonator is analyzed. Modulating the incoming laser intensity at a suitably chosen frequency, the mechanically induced oscillation of the reflected light is coherently downconverted to a frequency within the detection bandwidth. Additionally, based on the mechanical nonlinear response, the optical signal can be quantitatively converted into displacement, yielding a sensitivity of 7 $\mathrm{pm}/\sqrt{\mathrm{Hz}}$ at optical power levels of 20 μW. We detect and image mechanical modes up to the seventh harmonic of the fundamental mode at 7.7 MHz. © *2009 American Institute of Physics*.
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The resonant motion of micro- and nanoelectromechanical systems is increasingly investigated. Their small masses, high quality (Q) factors, and high integrability make them equally interesting for fundamental research as well as applications in sensing and signal processing.[1,2] Optical setups are among the most sensitive ones for the detection of the mechanical motion. With decreasing dimensions and increasing resonance frequencies of the mechanical systems the detection of the motion requires increasingly complex setups.[3] In particular, sensitive optomechanical transduction typically employs reference beams[3] and/or optical cavities.[4] These approaches equally require very stable lasers and optical paths. We employ a simpler fiber-optical setup, as sketched in Fig. 1 and described, e. g., in Ref. 5.

In this setup the sample is illuminated with light coming out of a bare close-by glass fiber and the scattered light is collected with the same fiber without additional optical components. Our investigated nanomechanical system consists of a stretched SiN wire[6] of dimensions 35 $\mu\mathrm{m}\times 250$ nm $\times 100$ nm, length, width, and height, respectively. The mechanical actuation is induced by dielectric forces caused by an essentially spatially inhomogeneous electrical field generated by suitably biased electrodes close to the resonator as discussed elsewhere.[7] Since the motion of the resonator only weakly modulates the reflected laser intensity, significant amplification of the detected signal oscillating at radio frequency of the mechanical resonances is required. Typically, amplifiers exhibit a trade-off concerning bandwidth, amplification factor, and amplifier noise. The photodiode with an integrated preamplifier (Thorlabs PDA55) used for this work has variable gain and bandwidth (maximum of 10 MHz). The datasheet shows that these quantities are approximately inversely proportional whereas the amplifier noise is rather constant with varying bandwidth. In order to exceed the amplifier constraints, we introduce a modulation of our laser intensity at frequency f_{RF}, as sketched in Fig. 1. Here, a square-wave modulation of the laser intensity is implemented with a homemade switching circuit. We actuate the mechanical resonator with frequency $f_{\mathrm{RF}}-f_{\mathrm{LO}}$. This driving signal is coherently generated by mixing the signal that modulates the laser with the signal of a local oscillator, operating at fixed frequency $f_{\mathrm{LO}}=0.9$ MHz employing a homemade single sideband modulator. Consequently, the light reflected from the driven mechanical resonator contains frequency components at the sum $(2f_{\mathrm{RF}}-f_{\mathrm{LO}})$ and difference (f_{LO}) frequency. The sum frequency typically exceeds the bandwidth of our detector and is suppressed. However, the difference frequency is coherently detected using a network analyzer. Sweeping f_{RF} while retaining $f_{\mathrm{LO}}=0.9$ MHz yields the frequency-dependent response of the mechanical resonator at $f_{\mathrm{RF}}-f_{\mathrm{LO}}$. In the following all experiments are performed at room temperature and a pressure below 5×10^{-4} mbar.

A typical response curve can be seen in Fig. 2(a); fitting a Lorentzian line shape yields the mechanical resonance frequency and quality (Q) factor. With the stroboscopic detection scheme, we are able to investigate also harmonic modes

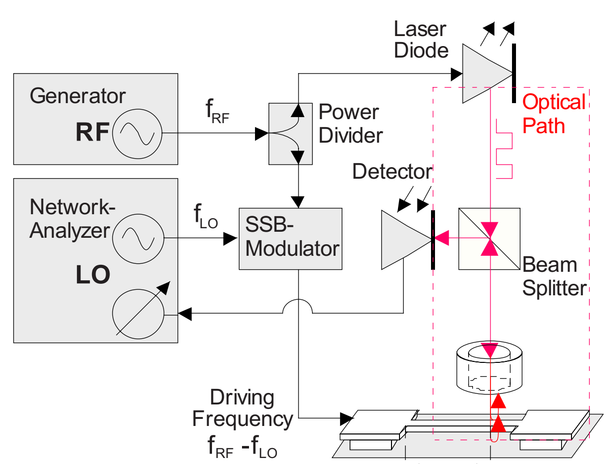

FIG. 1. (Color online) Schematical transduction setup; the electronic path is plotted in black; the area surrounded by the dashed rectangle depicts the optical path; arrows indicate the direction of signal propagation. The sample containing a string as nanomechanical resonator is mounted in vacuum just below the end of the optical fiber as indicated. As the sample is actuated at $f_{\mathrm{RF}}-f_{\mathrm{LO}}$ and the illuminating laser intensity is modulated at f_{RF} a coherent, low-frequency beat at f_{LO} is created on the photodetector.

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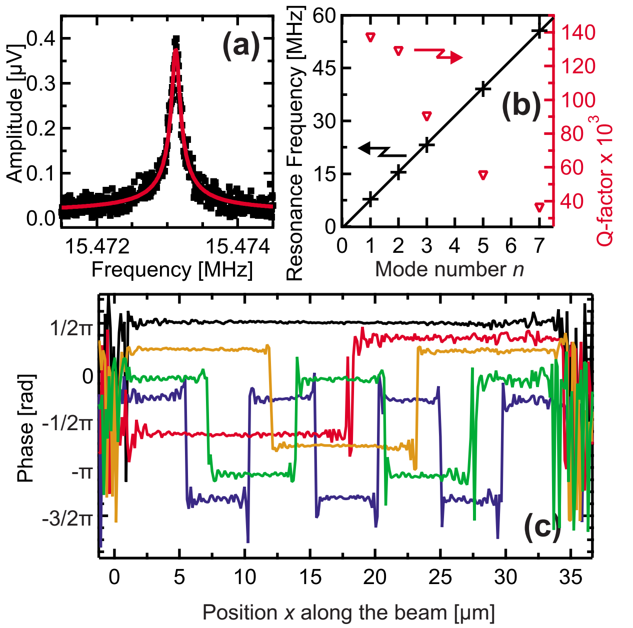

FIG. 2. (Color online) (a) Mechanical response and Lorentzian fit of a nanomechanical stretched SiN wire with dimensions 35 μm $\times$ 250 nm $\times$ 100 nm length, width, and height, respectively, driven around the second harmonic mode. (b) Resonance frequencies and quality factors of the fundamental mode and all observed harmonics are plotted vs mode number n, reflecting the number of antinodes along the length of the wire. To emphasize the scaling behavior of the frequencies a linear fit is shown. (c) Spatial distribution of the phase of the observed mechanical modes as measured with the detection fiber moved by position x along the wire (see Fig. 1). For clarity the curves are offset in phase with respect to each other.

of our mechanical resonator at frequencies beyond the bandwidth of our photodetector. The ability to record several modes has been demonstrated to be advantageous for sensing.[8] In Fig. 2(b), resonance frequencies and quality factors are displayed versus the respective mode number n corresponding to the number of antinodes along the length of the resonator. In contrast to a doubly clamped beam,[9] the frequencies can be clearly seen to scale linearly with mode number. As the implementation of the employed actuation scheme is spatially symmetric, we are not able to excite all antisymmetric, even n, modes. From the resulting spectrum, one can deduce that the model of a string can be safely employed to describe the resonant motion. The relatively large Q values are found to decrease with increasing frequency, a phenomenon generally observed[6] but still not quantitatively understood. In Fig. 2(c), we scan the detection fiber along the wire and plot the locally obtained phase of the oscillation; a method for convincingly identifying a given mode. It is noteworthy that techniques relying on modulation of the driving amplitude are not able to retrieve this information, see for example Ref. 10. Using a direct detection scheme of the mechanical resonance under cw illumination, we obtain a somewhat higher displacement resolution (yet bandwidth limited) and are able to measure the Brownian motion of the fundamental mode at 300 K. This enables us to quantitatively convert the measured signal into displacement.[11]

In the following, we describe how the nonlinear behavior of the resonator can be employed to transfer this displacement calibration to the higher harmonics. Related experiments have been reported in Ref. 12. At large driving amplitudes, the restoring force $F(z)$ exhibits nonlinear terms

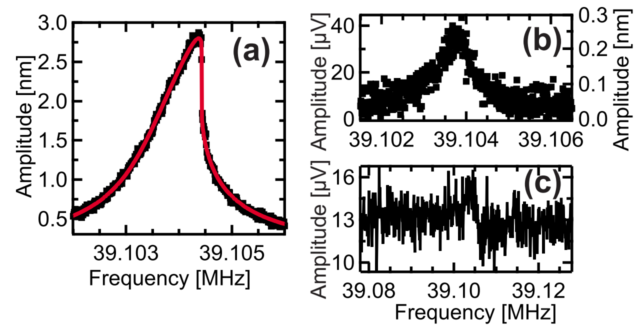

FIG. 3. (Color online) (a) Nonlinear response of the fifth harmonic mode; experimental data (black) and theoretical fit (red). The fit is employed to convert the detected signal into mechanical displacement. [(b) and (c)] Comparison of signal transduction using stroboscope (a) and (b) and cw illumination (c) measuring with 50 Hz bandwidth. Note that the driving amplitude in (a) and (c) are identical; therefore the noise floor in (c) can be estimated to be about 2 nm and is substantially larger than the one of about 50 pm (b) achieved in the stroboscopic detection scheme.

in displacement z. For convenience we write $F(z)=k\cdot z+m_{\text{eff}}\alpha_3\cdot z^3$. Here m_{eff}, $k=m_{\text{eff}}(2\pi f)^2$, and $m_{\text{eff}}\alpha_3$ denote the effective mass, linear spring constant, and cubic contribution, respectively, of the mechanical mode considered. For the case of a string, the spatial modes are described by a cosine, thereby a simple calculation yields the restoring force up to cubic order. We define L as half the wavelength of the resonant mode, which for the fundamental mode equals the length of the string l and for the higher modes $L=l/n$ with $n=2,3,\ldots$. With E, σ, and ρ being Young's modulus, tensile stress, and density of the resonator material, we obtain

$$\frac{F(z)}{m_{\text{eff}}}=\frac{\pi^2\sigma}{L^2\rho}z+\frac{(E+3/2\sigma)\pi^4}{4L^4\rho}z^3. \tag{1}$$

We note that this result reflects the well-known fact that a string doubles its resonance frequency when halving its length. Assuming a density of $\rho=3000$ kg/m^3,[13] the measured frequency of the fundamental mode translates into a tensile stress of $\sigma=830$ MPa, significantly less than the given specifications of the unprocessed SiN films (1400 MPa). For the amplitude conversion we note that the nonlinear term scales with the length as $\alpha_3\sim L^{-4}$. The differential equation employing the nonlinear restoring force, the so-called Duffing equation, can be solved in the case of a steady-state oscillation, yielding a frequency-dependent amplitude $|z|=|z|(f)$. The explicit calculations are not presented here and can be found for example in Ref. 14. With increasing actuation amplitude, the initial Lorentzian line shape begins to bend over to one side and eventually becomes bistable, a nonlinear phenomenon often seen in nanomechanics.[15] We fit the measured resonance curves near the onset of bistability with the solution of the Duffing equation. For the fundamental mode we thereby derive an absolute value for α_3. With the given geometry this translates into a Young's Modulus of $E=100$ GPa, reduced with respect to the literature value of spatially homogeneous SiN films around 300 GPa.[13] Applying the obtained values, continuum mechanics predicts a flexural contribution to the restoring force less than 2%.[6] To extend the calibration to the case of the harmonics, the values for α_3 are rescaled to obey $1/L^4$ scaling with respect to the fundamental mode [see Eq. (1)]. Thus a scaling factor is obtained to convert the measured detector signal into displacement. Figure 3(a) shows such rescaled measurement and the corresponding nonlinear fit. The fitting procedure proves to be surprisingly stable, even in the case that none of the fitting parameters, such as f_0, Q are held fixed. In contrast, if we directly employed literature values for E, appropriate for our wafer material, the conversion would yield amplitudes that are approximately half as large.

Using above calibration, we are able to determine the sensitivity of our setup. Figure 3(b) shows a weakly driven resonance detected in the stroboscopic mode, from which one can deduce a noise floor of about 50 pm measured at a bandwidth of 50Hz. This corresponds to a sensitivity of 7 $\mathrm{pm}/\sqrt{\mathrm{Hz}}$ at an average incoming laser power of 20 μW. The values obtained for the other modes deviate slightly from this value; we attribute this to the modulation amplitude of our laser that is measured to be not completely constant over the whole range of modulation frequencies (data not shown). Normalized to the laser intensity, these sensitivity values are comparable to those reported in Ref. 5 based on cw illumination. To quantify the signal-enhancement caused by the stroboscopic downconversion technique, Fig. 3(c) shows the same resonance as Fig. 3(a) employing cw illumination. The obtained maximum in the spectrum therefore corresponds to an amplitude of 3 nm, whereas the noise floor translates into an amplitude of at least 2nm rms, about a factor of 50 worse than in Fig. 3(b).

Assuming that the modulation of the illuminating laser at higher frequencies does not increase the noise of the optical signal detected at frequency f_{LO}, the detected signal decreases only in proportion to the mechanical displacement of the string and the sensitivity is expected to be independent of frequency.

Other stroboscopic techniques have been reported that do not require additional optical components: Ref. 16 employs short light pulses freezing the mechanical motion that produce phase-shifted spatial images and typically achieves displacement resolutions not exceeding nanometer. Another recently reported time-domain technique,[17] employing local interferometric effects, is equally based on short probing and excitation pulses. There, displacement resolutions in the picometer regime are observed up to resonator frequencies in the GHz regime. However, as pulsed excitation is applied, this technique imposes challenges when studying steady-state phenomena.

In conclusion we demonstrate a simple stroboscopic technique to coherently detect nanomechanical motion, separating the detection frequency f_{LO} from the frequency of mechanical motion $f_{\mathrm{RF}}-f_{\mathrm{LO}}$. This allows to far exceed bandwidth limitations imposed by sensitive detection electronics. Based on this approach sensitivities of the mechanical displacement down to 7 $\mathrm{pm}/\sqrt{\mathrm{Hz}}$ are demonstrated at an average incoming laser power of only 20 μW and resonant motion between 7 and 55 MHz. It is expected that this technique facilitates detection ranging in the GHz regime. Nonlinear characteristics of the mechanical resonator are employed to quantitatively convert the measured signal into displacement.

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