Doppler Optomechanics of a Photonic Crystal

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A laser beam directed at a mirror attached onto a flexible mount adds friction to its mechanical motion by the Doppler effect. For a normal mirror the efficiency of this radiative Doppler friction is very weak and practically masked by laser shot noise. We find that it can become very efficient using a photonic crystal mirror near its photonic band gaps. As an example, a Bragg mirror used at the long wavelength edge of its band stop can be efficiently optically cooled using the Doppler friction. The opposite effect opens new routes for optical pumping of mechanical systems: a laser pointing at a Bragg mirror and tuned at its short wavelength edge induces amplification of the vibrational excitation of the mirror leading eventually to its self-oscillation. These new effects rely on the strong dependency of a photonic crystal reflectivity on the wavelength.

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In a visionary paper of 1967, Braginski and Manukin [1] predicted that a mirror in motion relative to a light source would not only be subjected to radiation pressure but would also experience a less-known force opposing its velocity, namely, radiative viscous friction [2]. This velocity dependent force is due to the Doppler effect and to illustrate it they imagined a perfectly reflecting mirror attached to a lossless spring oscillating back and forth facing a light source of constant illumination power P_0. They found that the mechanical oscillations of the mirror-spring system would damp dissipatively at a rate $2P_0/mc^2$. Practically, however, this damping rate is exceedingly small even for the smallest diffraction-limited sized mirrors of mass m in the range of 10^{-15} kg [3] and it is not surprising that such a fundamental radiative damping has not yet been detected.

However, a point missed so far in the literature is that radiative damping is necessarily accompanied by fundamental fluctuations in radiative pressure due to the photon's statistical fluctuations [4]. So in reality two effects oppose each other. The larger the laser power, the larger the Doppler damping of the mirror oscillations but at the same time the larger the mirror fluctuations. In this work we determine that an equilibrium between damping and fluctuation is theoretically but impractically obtained at infinite laser power. In such an equilibrium we find that the mirror-spring mechanical oscillator fluctuates at an effective Brownian temperature of $h\nu/2k_B$, which would be typically few thousands of kelvin for a photon frequency ν in the visible or near-infrared range. In fact, even using a powerful shot noise limited laser source, Doppler friction is practically not detectable even for a perfect mirror with near unit reflectivity and this unless $2P_0/mc^2$ approaches the natural damping rate of the spring. Even for the smallest mirrors masses m of 10^{-15} kg and a record-low spring mechanical damping rate of 1 Hz, such a condition would necessitate laser fluences of 200 $\mathrm{MW/cm^2}$, a power density that would destroy even the best highly reflecting mirrors. In this work we show that by using a photonic crystal in place of the mirror and exploiting the very strong wavelength dependency of the reflectivity near its photonic band gaps, the radiative friction is not only amplified up to 5 orders of magnitude but it can also have its sign reversed to become a negative radiative damping. In this context, a negative damping is to be understood as a source of mechanical energy extracted from light that feeds the oscillating photonic crystal opposing in this way the natural damping of the spring. This work provides a method used to reveal the elusive fundamental Doppler friction of a moving mirror as well as a way of finding out if a negative Doppler friction can be measured.

A laser beam at wavelength λ and power P illuminates the mirror of reflectivity $R(\lambda)$ as schematized in Fig. 1. The

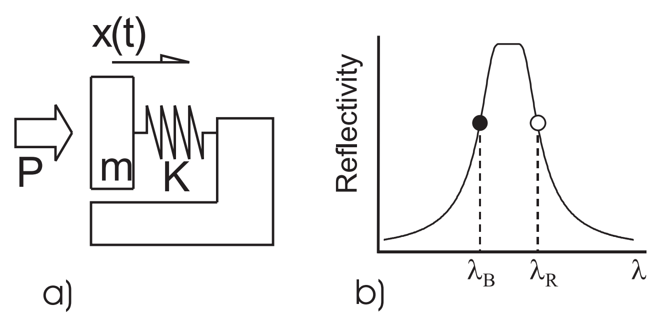

FIG. 1. (a) Schematics of a mirror of reflectivity $R(\lambda)$ and mass m mechanically attached and thermally anchored to a spring of rigidity K. The friction mechanisms internal to the spring are responsible for losses of mechanical energy at a rate Γ. A laser of power P is directed at the mirror from the left. (b) Schematics of the reflectivity wavelength dependency of a Bragg mirror. The gradients of R are maximized at the edges λ_B and λ_R of the mirror band stop.

radiation pressure acting on a moving mirror varies in proportion to the Doppler shift. We show here that because of this dynamic, the Brownian fluctuation of the suspended mirror loses its energy to the electromagnetic field. We find that the efficiency of Doppler cooling can be much increased using a mirror with a large and negative gradient of $R(\lambda)$. We also show that an amplification of the vibrational excitation can be reached for positive large enough gradient of $R(\lambda)$. Large gradients of reflectivity are found at the edges of the band stop of a Bragg mirror or more generally near the photonic band gaps of a photonic crystal.

The movable mirror of mass m is subject to a force $F_{\rm ph}$ due to radiation pressure. Its position x and velocity v of the center of mass of the mirror obey Newton's equation of motion, namely, $m(dv/dt) + m\Gamma v + Kx = F_{\rm ph} + F_{\rm th}$. The Langevin force $F_{\rm th}$ is introduced here in order to account for the Brownian fluctuation of the center mass of the mirror coupled to a thermal bath at temperature T. The damping rate Γ of the mirror mechanical fluctuation is a factor characteristic of the spring holding the mirror. The momentum transferred to the mirror per photon is given by $\hbar(k_0 - k_R)$. Here the incoming photons have their wave vector $k_0 = 2\pi/\lambda$ and the reflected ones have k_R oriented in the opposite direction. In the reference frame of the laboratory, when the mirror moves away from the light source, the reflected photons have their momentum reduced by Doppler effect such that $k_R = -k_0(1 - 2v/c)$. The radiation pressure is given by the rate of photon momentum transfer to the mirror and is also reduced by Doppler effect such that $F_{\rm ph} = RdN/dt\hbar(k_0 - k_R)$, where dN/dt is the number of impinging photons per unit time. The dependency of the radiation pressure on v is $F_{\rm ph} = 2R(dN/dt)\hbar k_0(1 - v/c)$, and by making use of the laser power $P = \hbar k_0 c(dN/dt)$ it is also $F_{\rm ph} = (2RP/c) \times (1 - v/c)$. In addition, when the mirror reflectivity is a function of λ, as in the case for a Bragg mirror operated near its band-stop edge, R also depends on the mirror velocity through the Doppler effect. In this case, we expand $R[\lambda(v/c)]$ in the experimentally relevant limit of $v/c \ll 1$. To the first order in k, the reflectivity is $R(v/c) \simeq R_0 + (v/c)\lambda(dR/d\lambda)$, so in the same way, to the first order in v/c the radiation pressure is $F_{\rm ph} \simeq (2R_0P/c)[1 - v/c + (v/c)(\lambda/R_0)(dR/d\lambda)]$. Using this expression in the equation of motion and grouping the velocity terms together we obtain an effective Newton equation of motion $m(dv/dt) + m\Gamma_{\rm eff} v + Kx = F_{\rm ph,0} + F_{\rm th}$ with a constant radiation pressure $F_{\rm ph,0} = 2R_0P/c$ and a Doppler modified damping rate

$$\Gamma_{\rm eff}/\Gamma = 1 + (2R_0P/mc^2\Gamma)[1 - (\lambda/R_0)(dR/d\lambda)]. \quad (1)$$

The constant force $F_{\rm ph,0}$ only shifts the average position of the center of mass and will be ignored in solving the effective equation of motion. For $dR/d\lambda \leq 0$, the optical contribution to the effective dissipation term $\Gamma_{\rm eff}$ takes energy away from the mechanical Brownian fluctuation and turns it irreversibly into electromagnetic energy through the Doppler effect. This amounts to cooling of the Brownian fluctuations of the mirror. We now determine the temperature of the vibrational motion of the mirror. For a harmonic oscillator the equipartition theorem links the temperature to the time averaged amplitude $\langle xx^*\rangle$ of the Brownian fluctuation, namely, $(1/2)k_BT_{\rm eff} = (1/2)K\langle xx^*\rangle$. In experimental conditions it is not the temporal dependency $x(t)$ but rather its spectral distribution x_ω that is typically measured. The spectrum x_ω is in fact a Fourier transform of $x(t)$. A mathematically convenient property of Fourier transformation is that the time averaged value $\langle x(t)x^*(t)\rangle$ term equals the frequency averaged value $\langle x_\omega x_\omega^*\rangle$. In Fourier space Newton's effective equation is $(-m\omega^2 + im\Gamma_{\rm eff}\omega + K)x_\omega = F_{\rm th,\omega}$, from which we obtain the spectrum $x_\omega = (F_{\rm th,\omega}/m)/(-\omega^2 + i\Gamma_{\rm eff}\omega + K/m)$. Here, for a nonabsorbing mirror, the spectral decomposition $F_{\rm th,\omega}$ of the thermal fluctuation driving force does not depend on the light and is evaluated from the situation in the dark. A reasonable guess about the nature of the driving force $F_{\rm th,\omega}$ is that there is no preferred frequency for the thermal fluctuations in the range of the mirror mechanical vibrational frequencies. This is reasonable at a vibrational mechanical frequency much lower compared to typical phonon frequencies with high density of phonon modes within the mirror material (THz). Within this approximation we assume the spectral power density $S_{\rm th}$ of thermal excitation of the mirror to be frequency independent, such that for any given frequency window $d\omega$ the amplitude of the thermal driving force obeys $F_{\rm th,\omega}F^*_{\rm th,\omega} = S_{\rm th}d\omega$. Using the expression of x_ω given above we obtain after some algebra $(1/2)K\langle x_\omega x^*_\omega\rangle = \pi S_{\rm th}/(4m\Gamma_{\rm eff})$. The left-hand side of this equation is $(1/2)k_BT_{\rm eff}$ under the assumption of equipartition and we end up with $(1/2)k_BT_{\rm eff} = \pi S_{\rm th}/(4m\Gamma_{\rm eff})$. Since the spectral power density $S_{\rm th}$ is not light dependent, in the dark we also have $1/2k_BT = \pi S_{\rm th}/(4m\Gamma)$. Comparing both expressions we obtain $T/T_{\rm eff} = \Gamma_{\rm eff}/\Gamma$, showing that an increase in $\Gamma_{\rm eff}$ leads to cooling [5,6]. This conclusion is premature because thus far we have ignored the effect of photon shot noise. For a given laser intensity we need to include the fundamental shot noise power fluctuation that induces a corresponding shot noise in the radiation pressure and hence drives an additional vibrational fluctuation or an added vibrational temperature. This added fluctuation could counteract the Doppler cooling. This fluctuation force $F_{\rm shot}$ is very much analogous to $F_{\rm th}$, but is proportional to the square root of the laser power. Here $F_{\rm shot,\omega}F^*_{\rm shot,\omega} = S_{\rm shot}d\omega$ with a frequency independent power density $S_{\rm shot} = (2R_0/c)2(Ph\nu)/2\pi$, where $h\nu$ is the photon energy. Assuming no correlation between the Brownian and shot noise, we obtain the new prescription $(1/2)KT_{\rm eff} = \pi(S_{\rm th} + S_{\rm shot})/(4m\Gamma_{\rm eff})$ leading to the expression for the effective vibrational temperature:

$$T_{\text{eff}} = (T + pT_{\text{phot}})/[1 + p(1 - \nabla R)], \quad (2)$$

where we defined (i) the unitless reduced laser power $p = 2R_0P/(mc^2\Gamma)$, (ii) the reflectivity gradient $\nabla R = (\lambda/R_0)(dR/d\lambda)$, and (iii) an effective photon temperature $k_BT_{\text{phot}} = R_0h\nu/2$. The power dependent term in the denominator originates from the Doppler effect while the one in the numerator is due to the photon shot noise. We see that for a mirror with a constant reflectivity $dR/d\lambda = 0$, the Doppler effect tends to lower the effective temperature while the shot noise terms increase it. For such a mirror, the condition $T_{\text{eff}} < T$, namely for cooling, is only possible when $T_{\text{phot}} < T$ or $R_0h\nu/2 < k_BT$. For visible or near-infrared photons and for high reflectivity mirrors $R_0 \sim 1$, this condition cannot be satisfied at room temperature. For a mirror with a large and negative reflectivity gradient, however, such that $\nabla R < 0$, Doppler cooling of the vibrational mode becomes possible when $T_{\text{phot}}/(1 - \nabla R) < T$ or $R_0h\nu/[2(1 - \nabla R)] < k_BT$. This gives a stringent condition for the reflectivity gradient. At room temperature using 1 eV photons on a mirror with $R_0 \sim 0.5$, namely for $T_{\text{phot}} = 2900$ K, the condition on the reflectivity gradient would be $-\nabla R > 8.7$. Experimentally this can be obtained using a Bragg reflector and a photon wavelength tuned at the higher wavelength edge of the band stop of the reflectivity [Fig. 1(b)]. Values as high and negative as $(\lambda/R_0) \times (dR/d\lambda) = -10^5$ are in fact within experimental reach in the visible and near-infrared [7]. Doppler cooling saturates using large enough laser power at $T_{\min} = T_{\text{phot}}/(1 - \nabla R)$. With the numerical example above this would be 29 mK.

Another interesting aspect of Bragg mirrors is that they also offer the opportunity to set the gradient ∇R positive enough to reach $\Gamma_{\text{eff}} \leq 0$ in Eq. (1). In this regime the mirror gains energy and starts to self-oscillate under the illumination of a cw laser. It enters possibly in a regime of nonlinear dynamics similar to the one predicted by Marquardt and co-workers [8] for deformable Fabry-Perot cavities and observed for instance in microtoroid cavities [9]. This effect would be interesting to demonstrate in this context of a cavityless system. The gain condition implies using a laser power large enough so that $P/\Gamma > mc^2/[2R_0(\nabla R - 1)]$. The energy term P/Γ represents the laser power averaged over a mechanical relaxation time constant $1/\Gamma$. This energy is compared to mc^2, the relativistic energy of the mirror, so we anticipate already the Doppler induced optomechanics might be very weak for macroscopic mirrors. We recently prepared a gold mirror ($dR/d\lambda \sim 0$) mounted on a silicon nanolever [3] with a mass in the 10^{-15} kg range. For such a mirror and for $R_0 \sim 0.5$, the condition for self-oscillation would be reached when $P/\Gamma > 90/(\nabla R - 1)$. Using a damping rate $\Gamma \sim 10\ \text{sec}^{-1}$ and $\nabla R \sim 10^5$, this would imply using laser power of 90 mW in order to enter a regime of self-oscillation. In order to increase the Doppler effect we see from Eq. (2) that one needs to decrease the mass m. For a Bragg mirror, this can only be done within bounds because photons probe the material periodicity within a finite penetration depth. The mass cannot arbitrarily be reduced by thinning the material; at some point the reflectivity and its gradient will degrade. Also, the lateral dimensions of the reflector cannot be reduced much less than the diffraction limit [3]. In the visible range we anticipate that the smallest masses would be in the 10^{-15} kg range.

Now we compare the strength of Doppler cooling with cavity cooling established in Refs. [5,6,10,11]. A Fabry-Perot cavity of length L separating the mirrors and finesse $F > 1$ stores electromagnetic energy with a typical ring-down time constant $\tau \sim (F/\pi)\tau_0$, where $\tau_0 = L/c$ is the photon time of flight across the cavity. τ is the typical time the cavity needs to build or lose energy upon a sudden change in laser power or in mirror separation. The photon pressure acting on the mirrors is proportional to the stored power so that the force acting on the mirror near a cavity resonance is not only enhanced by the cavity but also retarded with respect to the mirror separation fluctuation. Retarded terms amount to velocity dependent force terms similarly to the Doppler effect discussed above. For a cavity with at least one of the two mirrors mounted on a spring, this retarded effect induces an optical modification of the mechanical damping rate and consequently a modification of the vibrational temperature in the same way developed above. In order to cool the vibrational fluctuations of the mirror, it is necessary to detune slightly the laser wavelength to the red with respect to the cavity resonance [5,6,10–15]. Optimum detuning $\delta\{L/\lambda\}$ is obtained on the maximum slope of the dependency of electromagnetic energy stored in the cavity with respect to wavelength or mirror separation. This is the case when $\delta\{L/\lambda\} = -1/(g\sqrt{3})$. The reverse effect, namely optomechanical excitation, is obtained for blue detuning. We establish the extremal effective temperatures

$$T_{\text{eff}} \simeq (T + pgT_{\text{phot}})/[1 \pm p(L/\lambda)g^3/(g^2\omega_0^2\tau_0^2 + 1)], \quad (3)$$

where $g = 2F/\pi$ is proportional to the cavity finesse F. The sign depends on the side of the detuning with respect to a cavity resonance. For a given vibrational frequency $\omega_0 = (K/m)^{1/2}$ it turns out that optimal cooling (or pumping) is obtained at the condition $\omega_0\tau \sim 1$.

In expressing Eq. (3) we made the reduced power $p = 2R_0P/mc^2\Gamma$ appear explicitly in order to make a direct comparison with Doppler cooling. The term in the denominator is usually much larger than unity and we see that cooling efficiency using cavity effects can be made stronger by a factor as large as $g^3(L/\lambda)$ than Doppler cooling. Already for a finesse as low as $g = 10$ and for a length $L = 20\,000\lambda$, the cavity cooling can be made 10^7 more efficient than direct Doppler cooling and this considering equal mirror masses, laser powers, and damping rates. The use of large finesses allows a significant amplification of laser cooling but at the same time imposes that the moving mirror be part of a Fabry-Perot cavity and this is not always convenient. We stress however the fact that all semiclassical models derived thus far do not include directly the Doppler effect. In this framework, while mimicking it, cavity cooling cannot be interpreted as Doppler cooling. For completeness we introduced Doppler effect in our formalism of cavity cooling and found that it gives rise to corrections to the cooling efficiency that are small for cavity finesses $g \gg 1$ [16]. Interestingly enough, we should also note that in the Hamiltonian approach developed by Law [17] to describe the mirror-cavity field interaction, the Doppler effect is automatically taken into account as a result of momentum conservation in the system. Using a similar approach in the case of a single mirror under laser illumination would help to provide a comparison between Doppler and cavity cooling in a quantum framework.

We finish this Letter on an estimation of the cooling power involved with Doppler effect.

In the dark, the mechanical fluctuation dissipates its thermal energy $k_B T/2$ per mechanical degree of freedom and this at a rate Γ. The dissipated power is therefore $(k_B T/2)\Gamma$ and is in equilibrium with the power that feeds the fluctuation. When the mirror reflects the laser light, the effective vibrational mode end temperature is T_{eff}. When the vibrational mode is cooled down to a temperature T_{eff}, the steady state heat load in the mirror is $(k_B T_{\mathrm{eff}}/2)\Gamma$. Consequently, in order to maintain a temperature T_{eff}, the optical cooling extracts energy from the fluctuations of the mirror position at a rate $P_{\mathrm{cool}} = k_B(T - T_{\mathrm{eff}})\Gamma/2$, which is always smaller than $k_B T\Gamma/2$. So the maximum cooling power is $k_B T\Gamma/2$ both for Doppler and cavity cooling. This is in the range of 10^{-18} W at room temperature for Γ in the 10^3 sec^{-1} range. This might appear as a very weak cooling power, but it can be efficient enough to cool the lowest energy vibrational modes of an elastically suspended mirror since such modes are generally weakly coupled to the thermal bath.

In conclusion, we presented a simple formalism for laser Doppler cooling of the center mass fluctuation of a mirror attached to a spring. This effect is very weak but can become sizable when the mirror reflectivity is made to depend strongly on the photon wavelength. We also showed that effective temperature obtained through cavity cooling, in a formalism that does not include Doppler effect, mimics direct Doppler cooling but with a cavity amplification factor which is proportional to the third power of the cavity finesse and can easily reach 10 orders of magnitude. The reciprocal effect of Doppler cooling, namely Doppler optical pumping of the mirror motion, was also predicted. Interestingly enough, we showed that the use of an appropriate Bragg mirror should allow overcoming the shot noise limit and use directly the Doppler effect to set the mirror motion into self-oscillation. This could provide a very simple and noninvasive method to optically pump the motion of tiny mechanical resonators used in high sensitivity force detection.

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[1] V. B. Braginski and A. B. Makunin, Sov. Phys. JETP **25**, 653 (1967).
[2] A. B. Matsko, E. A. Zubova, and S. P. Vyatchanin, Opt. Commun. **131**, 107 (1996).
[3] I. Favero, C. Metzger, S. Camerer, D. König, H. Lorenz, J. P. Kotthaus, and K. Karrai, Appl. Phys. Lett. **90**, 104101 (2007).
[4] The photons' fundamental fluctuations were recognized to play a role in the optomechanical dynamics of a cavity-mirror system in the late 1990s. I. Tittonen, G. Breitenbach, T. Kalkbrenner, T. Müller, R. Conradt, S. Schiller, E. Steinsland, N. Blanc, and N. F. de Rooij, Phys. Rev. A **59**, 1038 (1999); K. Jacobs, I. Tittonen, H. M. Wiseman, and S. Schiller, Phys. Rev. A **60**, 538 (1999).
[5] O. Arcizet, P. F. Cohadon, T. Briant, M. Pinard, and A. Heidmann, Nature (London) **444**, 71 (2006).
[6] A. Schliesser, P. Del'Haye, N. Nooshi, K. J. Vahala, and T. J. Kippenberg, Phys. Rev. Lett. **97**, 243905 (2006).
[7] Y. Tissot, H. Limberger (private communication).
[8] F. Marquardt, J. G. E. Harris, and S. M. Girvin, Phys. Rev. Lett. **96**, 103901 (2006).
[9] H. Rokhsari, T. J. Kippenberg, T. Carmon, and K. J. Vahala, Opt. Express **13**, 5293 (2005).
[10] C. Metzger and K. Karrai, Nature (London) **432**, 1002 (2004).
[11] S. Gigan, H. R. Böhm, M. Paternostro, F. Blaser, G. Langer, J. B. Hertzberg, K. C. Schwab, D. Bäuerle, M. Aspelmeyer, and A. Zeilinger, Nature (London) **444**, 67 (2006).
[12] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, Phys. Rev. Lett. **99**, 093902 (2007).
[13] I. Wilson-Rae, N. Nooshi, W. Zwerger, and T. J. Kippenberg, Phys. Rev. Lett. **99**, 093901 (2007).
[14] C. Genes, D. Vitali, P. Tombesi, S. Gigan, and M. Aspelmeyer, Phys. Rev. A **77**, 033804 (2008).
[15] A. Dantan, C. Genes, D. Vitali, and M. Pinard, Phys. Rev. A **77**, 011804 (2008).
[16] C. Metzger, I. Favero, and K. Karrai (to be published).
[17] C. K. Law, Phys. Rev. A **51**, 2537 (1995).