Evidence of a nanomechanical resonator being driven into chaotic response via the Ruelle–Takens route

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(Received 29 January 2002; accepted for publication 17 July 2002)

Nanomechanical resonators, of importance for signal filtering and transduction, are investigated within the limit of extreme nonlinear excitation. The Ruelle–Takens route establishes the transition of a system into chaos with *n* frequencies present. By providing a set of three sources, this transition from linear to nonlinear and finally to chaotic response can be traced, and it is found in our experiment. Knowledge of potentially chaotic behavior of nanomechanical systems is crucial for their application. Our resonator system has an overall length of 4 μm and a cross section of about 100 nm$\times$300 nm with natural frequencies of ~50 MHz. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1506790]

With the advent of micro- and, more recently, nanoelectromechanical systems (NEMS) a new class of devices has now been introduced for applications in wireless information processing in the frequency range of 0.1–2 GHz.[1] These devices combine the advantages of mechanical systems, e.g., applicability as sensor systems and robustness to electrical shocks, with the speed and large scale integration of silicon electronics. Especially, the use of silicon as the starting material for processing both conventional electronic circuits and also NEMS favors this recent development.

NEMS are mostly operated in the regime of linear response and the dynamics of the devices thus remain fairly well controlled. However, like for the classical pendulum the driving amplitude of a nanomechanical resonator can be enhanced, and the system might then be brought into the nonlinear or even chaotic regime. In the case of application of NEMS in sensing or switching signals, it is therefore quite important to determine the possibility and the specific kind of such a transition.

In the case of a dynamic system with a single frequency *f*, its trajectory returns to the same point after a time interval of $(2\pi f)^{-1}$. Within a topological context, this describes the compact space that is mapped to a circle S^1. Accordingly, a two-frequency system is mapped to an $S^1\times S^1$ space, the usual two torus, and finally for *n* frequencies present in the system, an *n*-torus describes the dynamics. The Ruelle–Takens (RT) route, or *n*-torus route, to chaos establishes that an *n*-torus, for $n>1$, under adequate perturbation becomes a chaotic strange attractor. Thus, excitation by multiple frequencies is the crucial point, since the probability of choosing a region of chaos in parameter space increases with the dimensionality *n* of the *n*-torus.[2,3] The RT route to chaos depends on the choice of system parameters, and naturally on the strength of the nonlinearity.

In the present work, we address the gradual transfer of a nanomechanical system into the realm of chaos. The system of freely suspended beam resonators allows multiple excitation and observation of its mechanical response at the same time. We built systems of three coupled beam resonators with dimensions of 2 μm$\times$100 nm and 1 μm$\times$100 nm, respectively, that were clamped on two points that have been characterized before[4] (see Fig. 1). The total thickness, including metallization, of the beam is ~300 nm, with a SiO_2 sacrificial layer of 400 nm. The center beam is denoted by A. The main advantage of such a beam is that it can be easily excited via magnetomotive driving,[5] while being balanced by

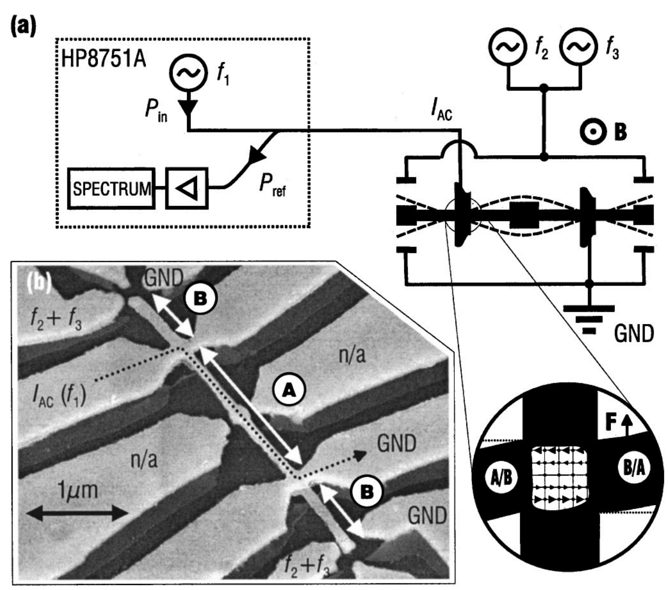

FIG. 1. (a) Experimental setup: Frequencies f_2 and f_3 are coupled capacitively to clapper resonators B. A network analyzer provides driving current I_{ac} at frequency f_1 and measures the overall response of the system for each set of frequencies (f_1, f_2, f_3). The inset shows schematically the mechanical coupling of the combined resonators, one driven by the force **F**. (b) Electron beam micrograph of the resonator system, B-A-B, and circuit diagram.

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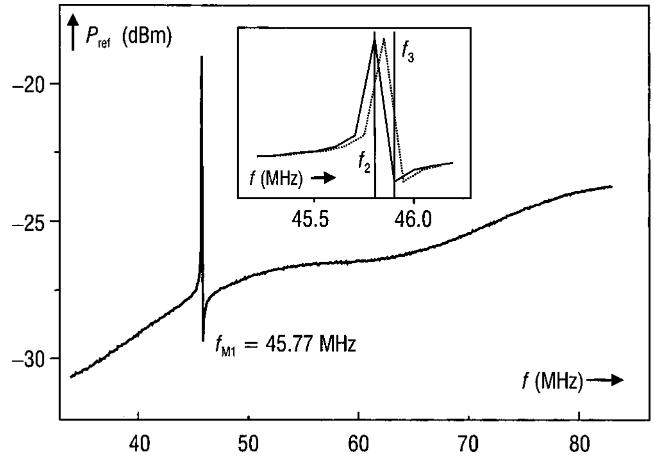

FIG. 2. Spectrum of mechanical excitation of base system A over a wide range of frequencies, with additional driving frequencies absent. The first mode detected is equal to the first harmonic mode of the center beam. The inset shows a magnification of the resonance peak in the range of interest. External excitations are then applied at neighboring frequencies, $f_2 = 45.802$ MHz and $f_3 = 45.892$ MHz, since those parametrically tune the base resonance to be at $\sim$45.847 MHz (dashed curve).

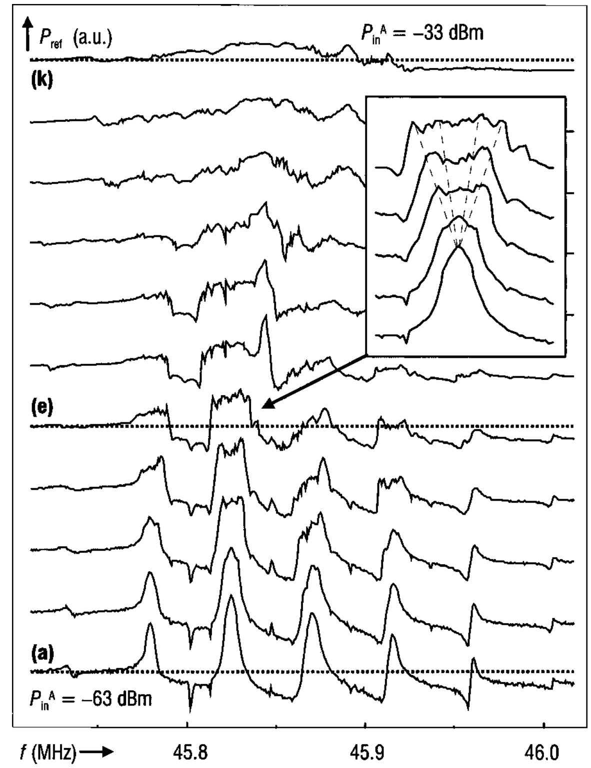

FIG. 3. Power spectrum vs scanning frequency f_1 for a rise in input power from −63 (a) to −33 dBm (k). Reflected power $P_{\rm ref}$ is given in arbitrary units, although it is always relative to a constant base line (dotted line). Due to the very low power of the third signal f_1 for curve (a) standard nonlinear mixing is observed. Peaks split and broaden with an increase in power $P_{\rm in}^{\rm A}$, marking the transistion from nonlinear to chaotic response (the inset is magnified).

the outer resonators. These so-called clapper resonators are denoted by B. As shown in Fig. 1(a), these clappers can be capacitively excited by nearby electrodes.[6,7] One electrode is biased with an ac signal at $P_{\rm in}^{\rm B} \approx +15$ dBm and the opposite electrode and the clapper tip itself are grounded. This excitation remains unchanged during the entire experiment. The crucial role of the resonator clamping points that separate the three beams is to reduce losses in a fashion similar to a tuning fork.[8]

Measurements are performed in a cryostat fitted with a superconducting magnet coil that provides magnetic fields up to $\mathbf{B} = 12$ T at 4.2 K. The mechanical response of the oscillating beam is determined by a network analyzer (HP87511A), scanning frequency f_1. The corresponding spectrum of mechanical excitation is shown in Fig. 2, in which the base mode of center resonator A is detected. Around this mode of frequency f_0 signal mixing is performed: excitations at f_2 and f_3 are coupled into the system via the two clapper resonators (B) by two Agilent 83650B(L) signal generators.

Increasing the current flowing through resonator A brings the entire system into the regime of nonlinear response. According to Ref. 9 application of two frequencies for sufficiently low power levels leads to standard mechanical mixing, as shown in Fig. 3. Traces for various values of $P_{\rm in}^{\rm A}$, ranging from −63 [trace (a)] to −33 dBm [trace (k)] are shown. Basically, the Fourier power spectrum versus scanning frequency for an increase in amplitude A_1 is given. The two mixing frequencies are centered at $f_2 = 45.802$ MHz and $f_3 = 45.892$ MHz, surrounding the system's natural frequency f_0. In case of ordinary nonlinear mixing of two input waves in a nanomechanical resonator the Fourier spectrum is determined by

$$\ddot{x}(t) + \mu \dot{x}(t) + f_0^2 x(t) + \alpha x^3(t) = A_2 \cos(f_2 t) + A_3 \cos(f_3 t), \tag{1}$$

where μ gives the attenuation coefficient, f_0 the natural frequency of the resonator, α the degree of nonlinearity, and $A_{2,3}$ and $f_{2,3}$ the amplitudes and frequencies of the two external signals, respectively. Using perturbation theory, the nonlinearity of the equation produces natural satellites at frequencies $f_0 \pm \Delta f$ and $\Delta f = f_2 - f_3$.

The present system of coupled resonators has somewhat more sophisticated properties, since the nonlinearity can be broadly varied. Furthermore, the system is composed of two resonator parts (A and B). For small values of A_i the dynamics of the resonator is described well by Eq. (1). As seen in Fig. 3(a) a satellite pattern similar to that in Ref. 9 is obtained. However, upon increasing the amplitude A_1, the coupled resonators B become more relevant by introducing new degrees of freedom. This is identified by the splitting of ordinary mixing into subsatellite resonances, which finally leads to a chaotic resonance pattern.

From a dynamical point of view, this experiment can be interpreted as a four-torus problem, since there are four independent frequencies in the system: the excitation frequency provided by the network analyzer f_1, the two input frequencies f_2 and f_3, and the natural frequency of the resonator $f_0' = f_0 + \delta(A_1)$ that is intrinsically nonlinear. The last, together with an increase in the nonlinearity, produces very different dynamics in the present experiment from in the previous one.[9] The signal spectrum in this experiment evolves from a distinct quasiperiodic one in Fig. 3(a) to a broad frequency spectrum Fig. 3(k) by changing the input power $P_{\rm in}^{\rm A}$ of frequency f_1. We interpret this phenomenon as a four-torus transition to chaos, extensively analyzed in nonlinear dynamics.[2,3,10]

Further numerical studies show that the strength of the nonlinear perturbation and the dimensionality of the torus are crucial in the process of breaking the torus towards a strange attractor.[3,11,12] Otherwise phase locking between different excitation frequencies can frustrate this transition.[3,10] Phase locking, however, implies a decrease in the number of peaks in the Fourier spectrum. As we observe in the spectrum, [e.g., in Fig. 3(e) and in the inset a broadening of the peaks and an increase in their number is found] we can conclude an *n*-torus→strange attractor scenario in the experiment.

In summary, it has to be pointed out that the chaotic region in parameter space of the *n*-torus increases with the dimensionality of the torus. This partially explains why chaos was not observed in earlier experiments,[9] which were perturbation of a three torus. The ingredients for the transition of *n*-torus to chaos, the four torus itself and strong nonlinearity, are present. Our interpretation of the NEMS resonator in the chaotic regime follows from analysis of the Fourier spectrum and theoretical predictions of the RT route. In addition to these, it is well established that the strength of the nonlinearity is the main parameter in the transition. Since the system for high amplitudes is already in the nonlinear regime, the addition of resonators B enhances the nonlinearity of the system and additionally contributes to the transition to chaos. This is supported even when considering a simplified model in which several resonance peaks are broadened with an increase in input power, eventually overlapping and finally competing without a distinct winning resonance.[13] In any case, the underlying topology of the system dynamics points unambiguously to the RT route to chaos.

The authors would like to thank J. P. Kotthaus for his continuous support and A. Kriele, K. Werhahn, and S. Manus for expert technical help. This work was funded in part by the Deutsche Forschungsgemeinschaft (DFG) [Bl-487/1 and Bl-487/2]. One of the authors (G.C.) thanks the Center for NanoScience for their hospitality and the Conselho Nacional de Pesquisa of Brasil (CNPq).

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