Nanomechanical resonators operating as charge detectors in the nonlinear regime

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Abstract. – We present measurements on nanomechanical resonators machined from siliconon-insulator substrates. The resonators are designed as freely suspended Au/Si beams of lengths on the order of 1–4 μ m and a thickness of 200 nm. The beams are driven into nonlinear response by an applied modulation at radio frequencies and a static magnetic field in plane. The strong hysteresis of the magnetomotive response allows sensitive charge detection by varying the electrostatic potential of a gate electrode.

The mechanical vibration of a violin string produces audible sounds in the frequency range of some 100 Hz to several 10 kHz. Halving the length of such a clamped string the eigenfrequencies are increased by an octave. Scaling down the string to only some 100 nm yields frequencies in the radio frequency (RF) range. Recent work on such nanomechanical resonators [1,2] demonstrated different schemes of detection and their versatility for applications in metrology [3–5]. Although radio frequency single electron transistors allow a more accurate charge detection [6], operating temperatures for these devices are extremely low ($\sim 45 \text{ mK}$) and the devices are very sensitive to electrical and thermal shocks. On the other hand, integration of robust and radiation-insensitive mechanically flexible structures with single electron devices on the nanometer scale offers not only high speed of operation but also broad tunability of the tunnel contacts. Applications of mechanical resonators in nonlinear oscillators [7,8] or parametric amplifiers [9] are of great importance for scanning probe measurements and accurate frequency counters or clocks in general.

In this work, we want to demonstrate how to build nanometer-sized mechanical resonators and how to apply the nonlinear response of these devices for charge detection. In the earlier approaches [3] torsional resonators were operated around 2 MHz strictly in the linear response regime. In our case the resonators employed have typical dimensions of only a few 100 nm in width and height and hence resonance frequencies up to 100 MHz. Applying a sufficiently large excitation amplitude, the suspended beam shows a highly nonlinear response, which in turn allows extremely sensitive charge detection with an accuracy of $0.1e/\sqrt{\text{Hz}}$. Moreover, the device represents a model to studying phenomena such as stochastic resonance and deterministic chaos in a mechanical system on the nanometer scale.

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Fig. 1 – Micrograph of the nanomechanical resonator used in the experiment: The gate couples to the resonator, machined out of Si with a 50 nm evaporated Au layer. Inset: close-up of the resonator applied in the measurements.

The material employed is a commercially available silicon-on-insulator (SOI) substrate with thicknesses of the Si layer and the sacrificial layer of 205 nm and 400 nm, respectively. Processing of the devices requires optical lithography in a first step by which a metallic Al/Au (180 nm) mask is deposited. Adjacently the nanostructure is defined by electronbeam lithography and deposition of an Al/Au layer with a total thickness of typically 80 nm. The metal layers deposited on Si during lithography are a thin adhesion layer of Ni/Cr (1.5 nm), a covering Au layer (50 nm), and an Al etch mask (30 nm). The sample then is dry-etched in a reactive-ion etcher (RIE) in order to obtain a mesa structure with clear-cut walls. Finally, we perform a hydro-fluoric (HF) wet-etch step to remove the sacrificial layer below the resonators.

The suspended resonator is shown in fig. 1: This particular beam has a length of $l = 3 \mu m$, a width of w = 200 nm and a height of h = 250 nm and is clamped on both sides (see inset of fig. 1). The gate contact couples on the complete length of the resonator. The remaining beam is a Au/Si hybrid for which different elastic moduli have to be taken into account in the numerical simulations (not shown here). From these and the measurements shown below we can conclude that the HF attacked the Si beam slightly, hence the resonance frequency is lower than usual. This enhances the nonlinear response of the beam, since the restoring forces are less rigid. This assumption is verified by the close-up of the suspended beam shown in the inset of fig. 1.

All measurements were conducted at 4.2 K in a sample holder with a residual ⁴He-gas pressure of about 10^{-2} bar. This ensures thermal coupling, while it naturally reduces the mechanical quality factor (defined as $\kappa = f/\Delta f$, where f is the frequency at resonance and Δf the half-width of the maximum). The sample is mounted between interconnecting microstrip lines, designed to feed the circuit with frequencies up to 10 GHz, and a magnetic field is applied perpendicular to the beam. The absolute resistance of the metal wire on top of the resonator was found to be 34 Ω , which results in an appropriate impedance matching of the circuit. The beam is set into motion by applying a high-frequency electromagnetic excitation in the range of 10–100 MHz and ramping a magnetic field in plane. This results in an effective Lorentz force generated perpendicular to the sample surface. When the radio frequencies applied match the mechanical resonance frequency of the beam, the oscillations dissipate the electromagnetic power and thus the measured reflected power at the detector is reduced. For excitation and final detection we use an HP 8594A spectrum analyzer. The hysteresis of the mechanical resonator is probed with an additional Marconi 2032 source, which can be ramped from lower to higher frequencies and vice versa. The preamplifier employed is a low-noise broad-band (UHF- to L-band) JS amplifier (MITEQ Corp. 1998) with a specified noise figure of NF = 0.6 dB and a gain of G = 30 dB.

The capacitive coupling between beam and gate (see inset of fig. 1) is determined by numerical evaluation with a commercially available program (MAFIA, electromagnetic problem solver, ver. 3.20). From these calculations we obtain a capacitive coupling between gate and beam in the linear regime of $C_{\rm gb} \cong 220 \,\mathrm{aF}$. The frequency shift δf of the mechanical resonance results from the capacitive coupling given by the electrostatic energy $E = Q^2/2C$, where Q is the accumulated charge on the gate and C the capacitance between resonator and gate. This term can be expanded with regard to the elongation amplitude u = u(t) of the suspended beam, which yields for the electrostatic energy with C = C(u) via a truncated Taylor expansion

$$E(u) = \frac{1}{2}\frac{Q^2}{C} \cong \frac{1}{2}\frac{Q^2}{C + \frac{1}{2}C''u^2} \cong \frac{1}{2}\frac{Q^2}{C}\left(1 - \frac{1}{2}\frac{C''}{C}u^2\right) = E - \frac{1}{4}\frac{Q^2}{C^2}C''u^2,$$
(1)

where $C'' = \frac{\partial^2 C(u)}{\partial u^2}|_{u=0}$ represents the second derivative of the capacitance with respect to the spatial coordinate u at u = 0. This gives with Q = CV a frequency shift of the mechanical resonance on the order of

$$\delta f = \sqrt{f^2 - \frac{C''}{2m_{\text{eff}}}V^2} - f \cong -\frac{C''}{4m_{\text{eff}}f^2}V^2,$$
(2)

where $m_{\rm eff}$ is the beam's effective mass (in our case $\sim 4.3 \times 10^{-16}$ kg) and V the applied gate voltage. It has to be noted that for an absolute charge measurement the necessary charging of all metallic contacts, *e.g.* bond pads and leads, has to be taken into account. For one of the bond pads, for example, we estimate a capacitance of $C_{\rm bp} = \epsilon A/d \cong 2.11$ fF. However, it is still possible to determine the relative charge δq on the closely connected gate with a high accuracy as will be shown below.

In fig. 2(a) the RF response of the beam is depicted: Applying a magnetic field in plane, we find an increase of the peak maximum proportional to $\sim B^2$ (plotted in the inset). The driving amplitude of the RF is -66 dBm, ensuring linear response of the suspended beam. The FWHM of the resonance is $\Delta f = (16 \pm 0.2)$ kHz, resulting in a mechanical quality factor at resonance of $\kappa = f_0/\Delta f = 2330 \pm 30$. We find a large discrepancy compared to macroscopic mechanical oscillators with $\kappa \sim 10^5$ [7]. This can be explained by the coupling gas in the sample holder and the fact that the surface tension in these small devices naturally has a larger influence than in macroscopic systems.

In fig. 2(b) the power coupled into the resonator is increased from -70 dBm to -50 dBmwhere we find a strong nonlinear response. In the present case the nonlinear response is identified by the distorted peak shape. Above a critical value of the excitation voltage the curve finally shows a bistability accompanied by a pronounced hysteresis. The transition occurs at about -53 dBm, although an asymmetry of the peak structure is found at -59 dBmalready. The nonlinearity is caused by the variation of the restoring force at the clamping points [1] and can be modelled by adding a cubic term in the equation of motion of the beam [7]. Comparing our data with a model derived earlier [8] we find excellent agreement (modelled traces are not shown).

