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Dynamic control and modal analysis of coupled nano-mechanical resonators

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We present measurements on nanomechanical resonators allowing full *in situ* tuning of their dynamic properties, including higher-order nonlinearities (up to fifth order) and the mechanical quality factor Q. This is accomplished by gating electrodes and balancing resonators, similarily to a classical tuning fork. A detailed modal analysis is performed and reproducibility of the device response is verified. Eigenfrequencies are in the range of 40 to 70 MHz, and quality factor rises up to $Q \sim 6 \times 10^3$. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1575491]

Ultrasmall mechanical systems have become an integral part of mesosocopic physics over recent years. Nanoelectromechanical systems (NEMS) are studied and manufactured to act as force detectors at the quantum limit,[1,2] probe mechanical quantum behavior,[3] and for communication applications,[4,5] to name only a few. However, limited tunability of these NEMS represents a certain challenge when it comes to implementation as an experimental detector or to sensor/filter application. In recent work,[6] we have shown the gradual transfer of a NEMS into a chaotic response. Here, we expand the study of nonlinear excitation of a similar device, including full dynamic and modal control.

The integration of multiple nanomechanical beam resonators is the main concept the device is based on: one doubly clamped central beam ($l \times w \times h$: 2 μm $\times$ 100 nm $\times$ 200 nm) is appended by two singly clamped cantilevers ($l = 1\,\mu$m) at each end, incorporating, similar to a tuning-fork, about half of the central beam's mass [see Figs. 1(a) and 1(b)]. We denote the outer cantilevers as so-called Q-tips, due to their crucial role in altering the system's quality factor Q. The entire system was manufactured from a Au-coated silicon-on-insulator chip, employing a sacrificial layer process, which we have demonstrated in detail elsewhere.[7]

In order to drive the resonator, the chip is mounted in a cryostat equipped with a superconducting solenoid, providing a magnetic field density up to 12 T and sample temperatures of $T = (1.5 - 100)$ K. An ac signal, generated by a network analyzer (HP-8751A), causes a sinusoidal Lorentz force in the wire due to the magnetic field. Displacement is detected via the reflected power P_{ref} as the system's impedance changes when excited in mechanical resonance.[8] Together with the resonator beams, we incorporated a set of tuning gates in the system. These gates, denoted by the letters A through F, enabled us to place specific parts of the resonator in an electric ac/dc field [see Fig. 1(a)].

We fabricated a series of resonators with equivalent layout, revealing similar spectra of mechanical excitation, as shown for a single structure in Fig. 1(c). Each peak in this spectrum corresponds to one eigenmode of the entire resonator M1, M2, and M3. In the case of a dc bias applied to a

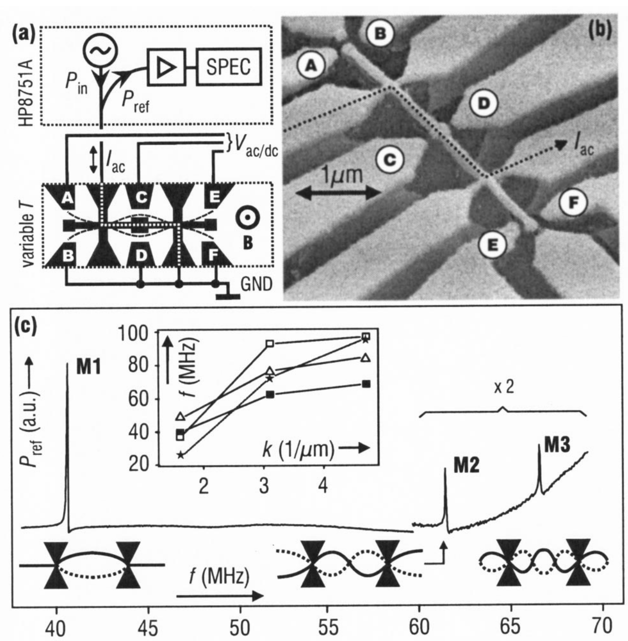

FIG. 1. (a) Experimental setup to measure power absorption by mechanical displacement; the ac-signal is generated and detected by an HP8751A network analyzer, which measures the ratio of incident and reflected power. (b) Electron-beam micrograph of the resonator system with the dotted line indicating ac-signal flow. (c) Spectrum of mechanical excitation for a magnetic field density $B = 12$ T and sample temperature $T = 4.2$ K, featuring the three main modes M1, M2, and M3. Inset shows the dispersion of three different samples and the respective FEM simulation ($\star$) for all observed modes.

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gate, we observe a shift of the resonance frequency following the square of the applied voltage V for ideal coupling of the resonator to the respective gate.[9] Applying separate gate voltages at gate pairs AB, CD, and EF, respectively, and observing the resulting shift in the resonator's natural frequency, allows one to determine the coupling strength of the specific part of the resonator for each eigenmode. In concert with both the simulation via finite element methods (FEM)[10] and analytic calculations, the experimental data allow one to determine the actual shape of each eigenmode M1–M3, as shown in Fig. 1(c). The double clamping, the presence of balancing cantilevers, and a favorable aspect ratio ($w<h$) strongly supports in-plane modal stability.

In the inset of Fig. 1(c), we show the dispersion of three of these systems for all observed modes; due to the particular "free-clamped-clamped-free" resonator configuration and mechanical attenuation by the clamping, a strong deviation from both straightforward beam ($\omega \sim k^2$) and string ($\omega \sim k$) mechanics is obtained. Corresponding modes and frequencies have been obtained via FEM simulations [see Fig. 1(c) inset].

Although energy is transferred from the driven central resonator to the Q-tips, the latter possess different eigenfrequencies. Their first-order resonance frequency is much larger than the excitation frequency of the central part. As a result, we observe virtually no displacement (via capacitive detection) of the outer Q-tips when the driven oscillator is in its lowest mode. From this and dc tuning experiments, we deduce the mode shape M1 to be actually as shown in Fig. 1(c), with an eigenfrequency of $f_{\mathrm{M1}} = 40.79$ MHz. Once excitation frequencies reach the first eigenfrequency of the Q-tips, displacement is detected. The resulting mode M2 corresponds to the second harmonic mode of the central system with $f_{\mathrm{M2}} = 61.24$ MHz.

A qualitative model of beam resonators can be obtained by using the Duffing equation. This equation models the displacement $y(t)$, where the restoring force of a driven and damped oscillator is expanded in a Taylor series:[11]

$$\partial_t^2 y(t) + 2\Gamma\omega_0 \partial_t y(t) + \sum_{\mathrm{odd}\ n}^{N} k_n y^n(t) = K\cos(\omega t), \qquad (1)$$

where Γ is the attenuation, ω_0 the (circular) eigenfrequency, ω the frequency of the driving force, and K its amplitude. In Eq. (1), we neglect asymmetric potential components.

In the linear regime of small displacements, a clear Lorentzian line shape is found, corresponding to the solution of the linear oscillator, only when $k_1 = \omega_0^2$ is nonzero in Eq. (1). Increasing the incident power P_{in}, the displacement becomes so large that orders up to k_3 have to be taken into account. The respective solution features a hysteresis,[12] which is reduced to the familiar sawtooth shape when excitation frequencies are swept monotonously. A positive $k_3 > 0$ renders a steep slope at the left side of the resonance, whereas a negative k_3 yields a jump at the right.

A more detailed analysis of the second-order resonance M2 revealed an even higher-order nonlinearity. In addition to a negative k_3, the Q-tips' motion causes a $k_5 y^5(t)$ term to appear. This corresponds to a sixth order term in the potential. Expansion in Eq. (1) up to $N=5$ yields up to five possible solutions for a single frequency. As a result, a kink in the resonance peak is observed (see Fig. 2).

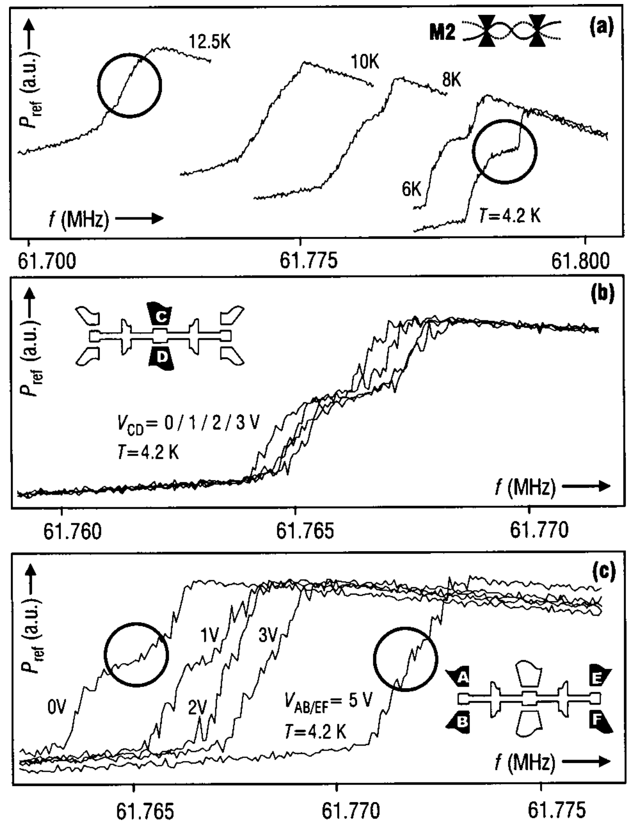

FIG. 2. Nonlinear excitation of the system in eigenmode M2 at a magnetic field density of $B=12$ T reveals a sixth-order term in the restoring potential. (a) Suppression of the kink by increasing bath temperature from 4.2 to 12.5 K. (b) Application of an additional electric potential across the gate pair CD up to $V_{\mathrm{CD}} = 3$ V leads to no substantial change of the mode M2 in nonlinear response. (c) Complete detuning of the kink by the electric potential across the outer gate pairs AB and EF occurs already at $V_{\mathrm{AB/EF}} \geqslant 2$ V. Different response for central and outer gate-pairs supports modal analysis of M2.

When tracing the peak at higher temperatures, the kink vanishes gradually. The resonator gains thermal energy, and both effective elasticity of the nanomechanical Si/Au configuration and the two clamping points substantially change, which, in turn, renders the influence of the k_5 nonlinearity less pronounced. The thermal detuning of the kink can be seen in Fig. 2(a) for sample temperatures $T = (4-12)$ K. Furthermore, the integrated tuning gates can be used to study the nonlinearity; applying an electric potential across the central gate pair CD does not influence the kink, as shown in Fig. 2(b), since in mode M2 the central displacement at the gate-pair is negligible. On the contrary, the kink can be entirely detuned, when the outer Q-tips are placed in an additional potential supplied by the gate pairs AB and EF [see Fig. 2(c)]. This gives further evidence for the strong influence of the clamping; that is, the central beam's k_5 can be detuned via the outer Q-tips.

The fundamental property characterizing a mechanical resonator is the quality factor Q, defined as the ratio of the oscillation energy ϵ_{osc} and the amount of energy which is dissipated ϵ_{dis} during one period:

$$Q = \frac{\epsilon_{\mathrm{osc}}}{\epsilon_{\mathrm{dis}}} = \frac{f_0}{2\Gamma}. \qquad (2)$$

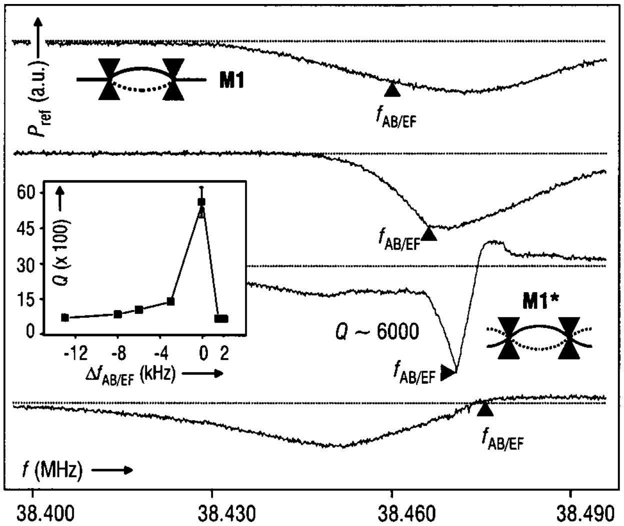

FIG. 3. Tuning of the quality factor Q by preparation of a high-Q mode M1* ($B=12$ T, $T=4.2$ K). The Q-tips are capacitively driven at frequencies $f_{\mathrm{AB/EF}}$. The best match occurs when maximum $Q\sim6000$ is attained. Reflected power P_{ref} is given in arbitrary units relative to reference (dotted) lines. The inset shows Q vs frequency mismatch of both systems (Ref. 15), normalized to the frequency of maximum Q. The increased error of the peak value $Q=6000\pm600$ is a result of the non-Lorentzian resonance shape.

For $Q\gg1$, the approximation $Q=f_0/\Delta f_0$ is valid, where f_0 is the eigenfrequency, and Δf_0 the full width at half-maximum of the resonance peak.

The present three-beam system is a design that minimizes energy loss;[13] this is achieved by adopting the concept of a tuning fork and adding electrodes gating the mechanical resonator, as shown. In addition to our previous work,[12] we employ the two outer Q-tips as "counterweights." The outer beams compensate for the central resonator's motion, rendering the center of mass fixed at all times. Consequently, we expect a decreased attenuation and an enhanced quality factor Q. In the classical tuning fork, two parallel fins ensure large Q.[14]

Apart from enhancing the quality factor Q, the present setup also allows us to tune this fundamental property of the system. As mentioned earlier, the clamping prevents the Q-tips from oscillating at base frequency. In this mode, compensation of the central resonator's motion is therefore suppressed. Hence, the relatively poor $Q\sim600$.[15] By ac-driving the gates, however, it is possible to prepare the desired mode shape.[7] The Q-tips are excited capacitively by applying a large ac signal at the outer pairs AB and EF. At the same time, magnetomotive excitation of the central beam is performed. When both frequencies match in a way that the whole resonator moves in phase,[12] the resonator's center of mass is fixed in space. Dissipation is reduced and the quality factor Q is strongly increased (see Fig. 3).

We have to point out that the resonance shape then clearly deviates from the usual Lorentzian. We explain this with an increased nonlinear behavior due to multiple excitation by two sources. Furthermore, the driving term in Eq. (1) must be replaced by $K_{\mathrm{A}}\cos(\omega_{\mathrm{A}}t)+K_{\mathrm{B}}\cos(\omega_{\mathrm{B}}t+\varphi)$. Consequently, we estimate[15] a maximum value of $Q\sim6000\pm600$ with an increased error.

In conclusion, we are able to fully control the dynamic response of a coupled nanomechanical resonator by featuring a design similar to a classical tuning fork. By additionally biasing the resonator with gating electrodes the specific mechanical modes can be identified, and nonlinearities can be detuned at constant driving power. Finally, Q-tuning within one order of magnitude is achieved by a combination of capacitive and magnetomotive excitation.

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[15] Quality factors Q have been obtained via a Lorentzian fit of experimental data and applying approximation of Eq. (2); that is, $Q=f_0/\Delta f_0$. Resulting errors are less than $2Q/100$ for Lorentzian behavior.