Optically tunable mechanics of microlevers

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We show how the mechanical rigidity of a slightly detuned miniature Fabry–Pérot cavity can be modified with light. We use a microcavity in which one of the mirrors is a soft compliant microlever optimized to detect bolometric forces. The static compliance can either be decreased to zero or increased considerably depending on the detuning of the light with respect to the cavity resonance.
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In a pioneering work, Braginsky and Manukin demonstrated that radiation pressure acting on the mirrors of a Fabry–Pérot (FP) cavity leads to a measurable coupling between the optical and mechanical properties of the resonator.[1] If the cavity is considered as a mechanical resonator, the properties of the resonator are modified by light. In fact, monochromatic radiation either increases or decreases the mechanical rigidity of the mirrors in the cavity depending on the detuning of the cavity from its optical resonance. The effect of photon pressure in a FP cavity was first experimentally demonstrated two decades ago.[2] The effect is important even for kilometer-long laser interferometers used as detectors of gravity waves.[3] This is because first, these interferometers are designed to be mechanically compliant along their optical axis, and second, the typical light power stored in the resonator is very large, typically hundreds of kilowatts.[3,4] This link between the optical and mechanical properties of a cavity is, in principle, scalable down to the micro- or nano-scale. In contrast to gravitational detectors, cavity lengths here are in the micrometer range but at the same time the rigidity of the mechanical systems is reduced by orders of magnitude allowing light-induced forces to play a significant role. Miniature mechanical resonators are used, for instance, in the detection of ultraweak forces[5] or in the tuning of surface-emitting semiconductor lasers.[6] We show here how we have tuned the mechanical properties of a microresonator with light. We do this by extending the concept of Braginsky to any light-induced forces, not just the radiation pressure. Experimentally, we find that the effective spring constant of our resonator can be decreased to negative values and also increased.

Our optomechanical device, sketched in Fig. 1(a), is in fact the force-sensing head of a specially developed low-temperature (4.2-K) scanning force microscope.[7] It consists of a gold-coated compliant silicon lever that forms one of the mirrors of a micro-FP cavity. The lever used in this experiment is a commercially available cantilever[8] with a nominal compliance $K\sim 0.1$ N/m. The other parallel mirror, located ~ 20 μm away, is the flat end of a monomode optical fiber (mode diameter 5 μm). The fiber is coated with a semitransparent gold film in order to increase its reflectivity as well as to allow for a fine-tuning of the cavity length. As shown in Fig. 1(b), this length is adjusted by applying an electrostatic potential U between both gold surfaces. The Si lever bends toward the mirrored end of the fiber under the effect of an attractive electrostatic force $F_{\mathrm{elec}}=-|\partial C/\partial z|U^2/2$, where C is the capacitance between both mirrors.[6] Since the variation of C is small over half a wavelength $\lambda/2$, we have $|\partial C/\partial z| \cong K\lambda/\Delta U^2$, where ΔU^2 is the period of the FP interferences, hence $-F_{\mathrm{elec}}/(K\lambda)\cong U^2/(2\Delta U^2)$. A laser beam of a 2-mW semiconductor diode ($\lambda = 673$ nm) is first attenuated by a continuously graded neutral density filter wheel and then transmitted through a polarizing beam-splitter cube before being launched into the 3-m-long fiber. At the other end, the light enters the FP cavity with a power P_0, which can be adjusted from a fraction of μW up to about 0.5 mW. The reflected light retraces its path through the same fiber, and is reflected by the beam-splitter cube onto a Si photodiode. The polarizing beam splitter rejects most of the spurious light reflected from the input end of the fiber, while the reflected signal from the cavity is maximized using a paddle polarization rotator mounted on a short portion of the fiber.

Our device is the miniaturized analogue of FP cavities with pendulum mirrors[2] used in interferometric gravitational

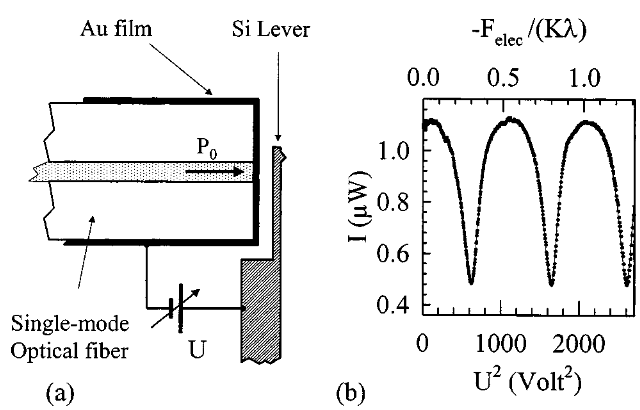

FIG. 1. (a) Schematics of the compliant micro-FP cavity (cavity length 20 μm). The fiber diameter is 125 μm and its mode diameter is 5 μm. The Si lever is Au coated. The potential U serves to tune the cavity length. (b) Reflected light as a function of U^2. Here, $K\lambda\sim 80$ nN. The full line through the data is the best fit of a FP reflectivity model.

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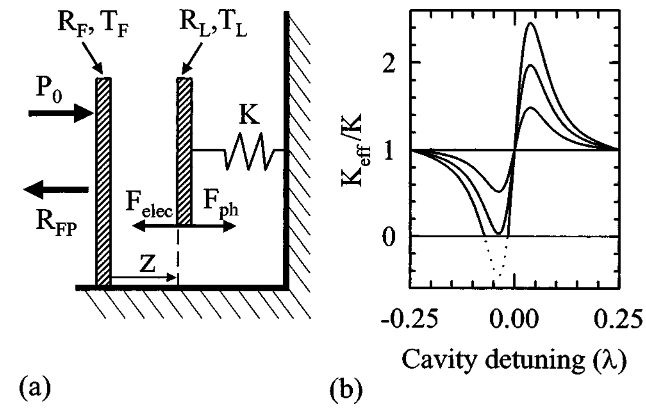

FIG. 2. (a) Optomechanical model for a microcavity with a compliant mirror. (b) Calculated effective spring constant as function of the cavity length centered on a FP resonance using $R_F=0.35$, $T_F=0.65$, and $R_L=0.55$. The curves with increasing amplitudes are calculated for photon-induced forces $F_0/(K\lambda)=0.025$, 0.05, and 0.075, respectively. The dotted part of the curve shows the region of bistable behavior for which $K_{\mathrm{eff}}<0$.

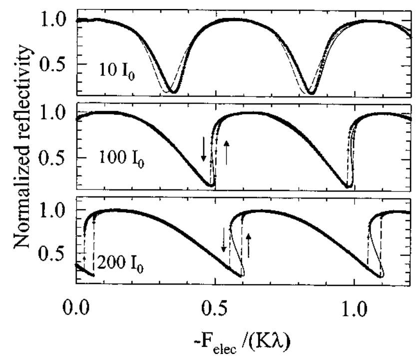

FIG. 3. Reflectivity data from a compliant microcavity as a function of the electrostatic tuning force for three different P_0. Here, $K\lambda\sim 80$ nN. The full line is the best fit to the model using $R_F=0.35$, $R_L=0.55$. (a) The dashed line is measured at very low photon power $I_0\sim 1\ \mu$W. (b) and (c) The dashed line connects the data points. The arrow shows the hysteresis direction.

wave detectors.[3,4] Figure 2(a) shows an equivalent optomechanical model for such systems. The photons filling the cavity exert a force on the lever. Assuming a monochromatic electromagnetic plane wave with a wavelength λ, it is easy to calculate the photon-induced force $F_{\mathrm{ph}}=F_0T_F/(4R)\ [g^2/(1+g^2\sin^2\phi)]$, which is proportional to the light intensity at the lever position. T_F is the transmission through the mirrored end of the fiber, and $\phi=2\pi z/\lambda$ is the phase determined by the separation z between both mirrors. The effective reflectivity $R=(R_LR_F)^{1/2}$ is determined from the lever and fiber reflectivity, R_L and R_F, respectively. The factor $g=2\sqrt{R}/(1-R)$ relates to the cavity finesse $\mathcal{F}=(\pi/2)g$, and F_0 is the force that the lever would experience in the absence of the cavity. F_0 is assumed proportional to P_0, and is not necessarily limited to the radiation-pressure force. In the present experiment we exploit bolometric forces. Such forces become dominant when the lever is designed to absorb part of the light and arise through the expansion of the thin Au layer on the Si lever. As for a bimetal blade, the lever bends with a deflection proportional to the amount of energy absorbed.[9,10] Consequently, the lever experiences a light-induced force that opposes the mechanical restoring force. As expected for a FP cavity, the strength of the light-induced force is a periodic function of z and peaks at cavity lengths that are an integer multiple of λ/2. The resulting effective spring constant of the lever in the FP configuration is to the first order in z, $K_{\mathrm{eff}}=K-\partial F_{\mathrm{ph}}/\partial z$, which is

$$\frac{K_{\mathrm{eff}}}{K}=1+\pi\left(\frac{F_0}{K\lambda}\right)\frac{T_F}{R}\left(\frac{g^2}{1+g^2\sin^2\phi}\right)^2\sin\phi\cos\phi, \quad (1)$$

which is a function that depends on z through the phase ϕ. The idea central to the force detection scheme presented here is based on the fact that $\partial F_{\mathrm{ph}}/\partial z$ can be made positive or negative, depending on z. Since the photon-induced force gradient and its sign are tunable by adjusting the light intensity and fine-tuning the cavity length z, K_{eff} can be softened continuously down to near 0 for positive gradients, or can be made stiffer than K by choosing a cavity length such that $\partial F_{\mathrm{ph}}/\partial z<0$. This behavior modeled by Eq. (1) and shown in Fig. 2(b) was first discussed by Braginsky[1] for the special case of radiation pressure. We focus now on the regime of cavity lengths tuned to minimize or maximize K_{eff}. In order to benefit from the analytical tractability of this model, we assume $g\gg 1$. This condition imposes a regime of FP cavity finesse $\mathcal{F}$ large compared to unity, a situation that is experimentally satisfied here. K_{eff} can be expanded near its region of minimum (maximum) values $K_{(-)}$ ($K_{(+)}$):

$$\frac{K_{(\pm)}}{K}\cong 1\pm\left(\frac{F_0}{K\lambda}\right)\frac{3\pi\sqrt{3}}{16}\,\frac{T_F}{R}g^3\left[1\mp 18\pi^2g^2\left(\frac{\delta z}{\lambda}\right)^2\right]. \quad (2)$$

The value $K_{(\pm)}$ is obtained when the cavity length is adjusted to $z_{N(\pm)}=(\lambda/2)[N\pm 1/(\pi g\sqrt{3})]$, close to the condition for the nth FP resonant mode. A minimum (maximum) is therefore obtained when the cavity length is detuned by a distance $\Delta=-(+)\lambda/(2\pi g\sqrt{3})$ from a resonance. The length $\delta z=z-z_{N(\pm)}$ in Eq. (1) is the detuning distance away from extremum $K_{\mathrm{eff}}=K_{(\pm)}$ conditions. Equation (2) shows that it is possible to adjust the value of the effective spring constant by adjusting the photon force F_0. A critical situation for which $K_{(-)}=0$ is obtained for $F_0=F_{\mathrm{crit}}=16/(3\pi\sqrt{3})(R/g^3T_F)K\lambda$. By way of example, this critical force would correspond to $F_{\mathrm{crit}}\approx 0.4$ pN for $R=T_F=0.5$, $K=10^{-5}$ N/m and $\lambda=1\ \mu$m. Relying on the radiation pressure alone, $F_0=2P_0R_L/c$, a modest power $P_0=130\ \mu$W is required to reach the critical force F_{crit}. When F_0 exceeds F_{crit}, the effective spring constant $K_{(-)}$ is negative, as seen in Fig. 2(b). Under such conditions, the lever is not in a stable equilibrium position and keeps moving under the effect of the photo-induced force until the effective spring constant becomes positive again. The lever can move into two new equivalent stable values of z, making the system bistable. The observation of such an optically induced bistability was evidenced for the first time two decades ago in an elegant experiment by Dorsel and coworkers[2] using a macroscopic FP cavity with a mirror suspended to swing as a pendulum.

Figure 3 shows the normalized reflectivity R_{FP} of the compliant microcavity system for three different values of input power. As P_0 is increased, the interference dips become very asymmetric. At the highest power, a hysteresis is measured, revealing a bistable behavior appearing at periodic values of F_{elec}. We modeled this behavior using the reflectivity of a FP cavity $R_{\mathrm{FP}}=1-T_LT_F/(4R)\ [g^2/(1$

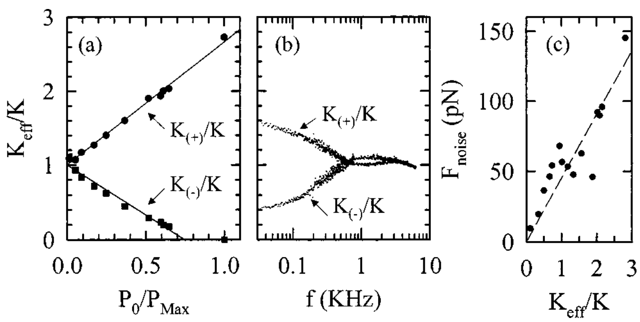

FIG. 4. (a) Change in the effective static spring constant as a function of the light intensity. The lower (upper) branch corresponds to a softening (stiffening) of the spring constant. (b) The corresponding frequency response measured at $P_0/P_{max}=0.3$. (c) Noise level in the detected force as a function of the spring constant at 300 K. The bandwidth of the measurement is 15 kHz.

$+g^2\sin^2\phi)]$, together with the lever equilibrium relation for the forces $F_{ph}+F_{elec}-Kz=0$, namely, $F_{elec}=Kz-F_0T_F/(4R)\ [g^2/(1+g^2\sin^2\phi)]$. By plotting R_{FP} against F_{elec}, we remove the explicit dependence on z. We used this procedure to fit very convincingly the measured asymmetries and hysteresis seen in Fig. 3. The force F_0 needed to account for the measured data is about a factor 10^3 larger than the radiation-pressure originating from photon momentum transfer, confirming that the effect measured here is purely bolometric. The pronounced asymmetries in Fig. 3 observed for larger P_0 are easy to understand. As seen in Fig. 2(b), the effective spring constant is larger than K for cavity lengths tuned just longer than a resonant length (i.e., positive detuning). In such a regime, a larger force F_{elec} must be applied in order to move the lever such that the reflectivity reaches a given value. The opposite is true when K_{eff} is made smaller than K for negative detuning lengths. The very fact that we measured a bistable regime in the reflectivity demonstrates that we were able to tune K_{eff} to negative values. The fit for different P_0 provided the $K_{(\pm)}/K$ is shown in Fig. 4(a), which is also $K_{eff}/K=[\partial R_{FP}(0)/\partial F_{elec}]/[\partial R_{FP}(P_0)/\partial F_{elec}]$. The effect of the optical tuning of K_{eff} disappears above a cutoff frequency of about 500 Hz as $K_{(\pm)}/K$ both tend toward unity, as seen in Fig. 4(b). Such a low frequency cutoff is consistent with a bolometric behavior, which has an inherently slow response.

The lowered effective spring compliance is not a constant, but rather a quadratic function of δz, as seen in Eq. (2). We have therefore to define a dynamic range Dz for the motion of the lever in which the measurement of force is possible. In terms of δK, the maximum tolerable variation of the spring constant for a given experiment, we determine the maximum available range for the lever motion amplitude $Dz=\lambda g^{-5/2}[(32/(27\sqrt{3}\pi^3)(R/T_F)(\delta K\lambda/F_0)]^{1/2}$. Enhanced sensitivity (i.e., larger F_0) is thus obtained at the cost of a reduced dynamic range. In addition, a reduction in the spring constant leads to an increased amplitude of thermal fluctuations in the lever position. We find that sensitivity to force increases faster than the noise, leading to a significant net improvement, as seen in Fig. 4(c). We determine the fundamental thermodynamic limits considering the lever to be tuned at its optimal sensitivity $K_{eff}=K_{(-)}$. The dynamic range cannot be chosen smaller than the amplitude of Brownian fluctuations $\langle z^2\rangle=k_BT/K_{(-)}$. Setting $\langle z^2\rangle=Dz^2$ leads to the minimum possible δK given by $\delta K_{noise}=(K/K_{(-)})\ (27\sqrt{3}\pi^3/32)\ (F_0/K\lambda)\ (T_Fg^5/R)\ k_BT/\lambda^2$. Making use of Eq. (2), we eliminate the term $F_0/K\lambda$ from this relation, so that $\delta K_{noise}=(3\pi g/\lambda)^2(K/K_{(-)}-1)(k_BT/2)$. Finally, we make use of the condition that $K_{(-)}\geqslant\delta K_{noise}$. Using this inequality in this equation, we determine $K_{(-)}/K\geqslant[(1+4\beta)^{1/2}-1]/2\beta$, where $\beta\ k_BT=(9/8)K(\lambda/2\pi g)^2$ is about the energy required to displace the lever over a half-width of a FP resonance. We see that at high temperatures such that $\beta\ll 1$, we get $K_{(-)}\geqslant K$, indicating that in this limit the reduction of spring constant is of no advantage. In the opposite limit $\beta\gg 1$, the inequality simplifies to $K_{(-)}\geqslant K/\sqrt{\beta}$, which shows that the lever rigidity can indeed be reduced. The minimum detectable force is limited by the Brownian force noise given by $F_{noise}=(K_{(-)}\ k_BT)^{1/2}$. Hence, for $\beta\gg 1$, $F_{noise}\geqslant(Kk_BT/\sqrt{\beta})^{1/2}$, which should be compared to $[Kk_BT]^{1/2}$, the nominal Brownian force noise floor. The optical tuning of the effective spring constant provides an improved sensitivity only if $[Kk_BT]^{1/2}>F_{noise}$. This can be expressed in terms of the maximum possible sensitivity enhancement factor $\eta=[Kk_BT]^{1/2}/F_{noise}$, namely, $\eta=\beta^{1/4}$. The condition $\eta>1$ indicates that optical reduction of the spring constant leads to a signal to noise ratio in the force detection enhanced by a factor η. A cavity with $g=3$, $\lambda=1\ \mu$m, and $K=10^{-1}$ N/m would give a maximum sensitivity enhancement of $\eta\sim 50$ at 4.2 K.

In conclusion, we have demonstrated that the rigidity of a microlever coupled to a FP cavity can be optically tuned. While the photon pressure is detrimental to the proper operation of a gravitational wave antenna we have shown instead that it can be used profitably for the quest for weak forces using microlevers. Making use of the photon pressure should open a fundamental new avenue for tuning the mechanical properties of small oscillators.

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