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LETTER TO THE EDITOR

Ballistic electron transport through a quantum point contact defined in a $Si/Si_{0.7}Ge_{0.3}$ heterostructure

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Abstract. We have studied the low-temperature ($T = 25$ mK) ballistic transport of electrons in a split-gate device fabricated from a $Si/Si_{0.7}Ge_{0.3}$ heterostructure. In the absence of a magnetic field the conductance is quantized in units of $i \times 4e^2/h$ as the gate voltage is tuned where i is the number of occupied subbands. In this system, both the spin and the valley degeneracies have to be taken into account. These can be lifted by applying a perpendicular magnetic field. Magnetic depopulation of the one-dimensional subbands is observed and can be simulated using a simple square well potential model.

Electron transport in quasi one-dimensional (1D) systems has been studied intensely during recent years [1]. Quantum point contacts (QPCs), i.e. constrictions defined in the plane of a two-dimensional electron gas (2DEG) with a width of the order of the electron Fermi wavelength and a length much smaller than the elastic mean free path, have proved to be excellently suited for the study of quantum transport phenomena. These have been realized in split-gate devices, for example, which offer the possibility to tune the effective width of the constriction and thus the number of occupied 1D levels via the applied bias voltage. One of the most striking results in this field was the discovery of the conductance quantization in units of $2e^2/h$ as the gate voltage is swept in the absence of a magnetic field [2, 3]. This is a manifestation of the fact that each occupied subband contributes an equal amount of $2e^2/h$ to the conductance G provided that the electrons travel ballistically, i.e. without scattering, through the constriction. Theoretically, the phenomenon can be readily explained by the Landauer–Büttiker formalism treating transport as a transmission problem [4, 5] and has been calculated in a variety of different theoretical treatments [6, 7].

By applying a perpendicular magnetic field the 1D subbands of a QPC are transformed into hybrid magnetoelectric subbands. Increasing the field strength results in the magnetic depopulation of the levels as the number of occupied levels is reduced, as shown in the experimental studies of van Wees *et al* [8] and Wharam *et al* [9]. The conductance of the QPC remains quantized since it is independent of the nature of the subbands, and the experiments have demonstrated a smooth transition from zero-field electric quantization through to the quantum Hall effect. By fitting the depopulation data to appropriate models [10] for the potential, information on the carrier density in the constriction and its width can be deduced.

Since high-mobility 2DEGs are required to observe conductance quantization most work so far has been conducted on GaAs/AlGaAs heterostructures. However, the quality of state-of-the-art $Si/Si_{0.7}Ge_{0.3}$ heterostructures has become sufficiently high over the last few years [11–13] and it is possible to realize 1D transport through periodic quantum wires in the quasi-ballistic regime [14]. In this letter we report on ballistic electron transport through a QPC which was defined in a $Si/Si_{0.7}Ge_{0.3}$ heterostructure. The sample we used was grown at Daimler-Benz by molecular beam epitaxy (MBE). The 2DEG is confined in a Si layer about 66 nm beneath the surface—deposited on top of a thick, relaxed SiGe buffer layer with graded Ge content—and possesses a carrier density $n_s = 4.45 \times 10^{11}$ cm^{-2} and an elastic mean free path of about 1.3 μm at low

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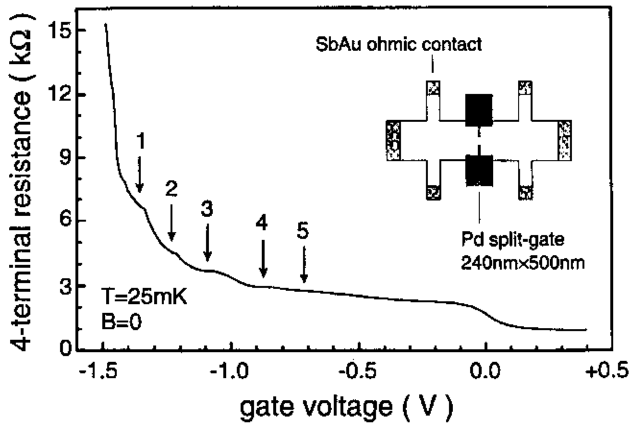

Figure 1. Four-terminal resistance as a function of the applied gate voltage at zero magnetic field. The arrows indicate steps resulting from conductance quantization in the point contact. Inset: schematic view of the device geometry. The voltage drop along the constriction was measured between probes on the same side of the Hall bar.

temperatures. Furthermore, the electrons occupy only the two subbands originating from the Δ valleys in the conduction band perpendicular to the plane of the 2DEG. This leads to a valley degeneracy $g_v = 2$ in addition to the spin degeneracy g_s and to an effective transport mass $m^* = 0.19m_0$. Samples from the same wafer were recently used to study the properties of field-effect induced electron channels using a modulated Schottky gate [15].

To fabricate the QPC a standard Hall bar was first defined by optical lithography and wet chemical etching and its side walls were coated with Si_3N_4. Ohmic contacts made of Sb and Au were then deposited and alloyed. Finally a Pd split-gate was defined between the Hall voltage probes by means of electron beam lithography. The constriction of this finger gate was lithographically 500 nm wide and 240 nm long. The sample geometry is schematically shown in the inset of figure 1. All measurements were conducted at $T = 25$ mK in a dilution refrigerator. Standard lock-in techniques were used with currents of 1 nA at 81 Hz.

Figure 1 shows the four-terminal resistance R_{4t} of the device as a function of the applied gate voltage V_{gate} at zero magnetic field. The voltage drop along the constriction was measured between two voltage probes on the same side of the Hall bar. R_{4t} therefore also contains serial contributions from the 2D regions adjacent to the point contact. As V_{gate} is reduced from +0.4 V one first observes a step around zero bias when the electron gas is depleted underneath the gates. At $V_{gate} = -0.1$ V, a short 1D channel has clearly been formed. The data on the depletion in this sample are consistent with the behaviour of bulk-gated Hall bars [15]. As V_{gate} is further decreased R_{4t} initially rises monotonically. This increase arises from the narrowing of the channel and the simultaneous reduction of the carrier concentration. 'Pinch-off' occurs around -1.6 V, but more significantly distinct, step-like features are clearly observable between -0.7 V and -1.4 V.

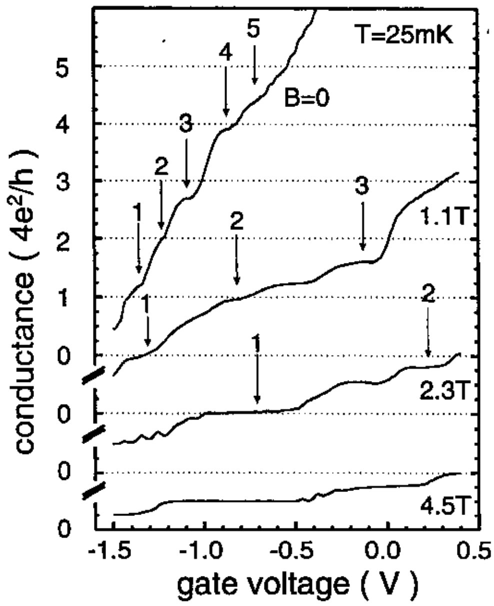

Figure 2. The conductance of the point contact at $B = 0$, 1.1, 2.3 and 4.5 T plotted as a function of the applied gate voltage. The curves are offset vertically for clarity. The shift of the conductance plateaus as marked by the arrows and the indices illustrates the magnetic depopulation of the 1D subbands. Additional steps of $2e^2/h$ and e^2/h appear when the spin and valley degeneracies are lifted by the magnetic field.

These plateaus originate from the conductance quantization in the point contact and are marked by the arrows in figure 1. To calculate the conductance G, a serial resistance of 1.3 kΩ arising from the 2D leads was first subtracted from R_{4t}. In figure 2, $G(B = 0)$, the uppermost curve, is plotted as a function of the applied gate voltage. Up to five subbands contribute to the conductance of the point contact. Note that the observed steps are close to a quantization of G in units of $4e^2/h$. This is a factor of two larger than the value reported in GaAs/AlGaAs heterostructures [2, 3] and is a manifestation of the additional valley degeneracy in the Si/SiGe system. Obviously, the carrier density-dependent valley splitting that has been observed in the similarly shaped, triangular potential wells of Si metal–oxide–semiconductor field-effect transistors (MOSFETs) [16, 17] is, at our carrier densities, too small to be resolved experimentally in the absence of a magnetic field. Similar conductance quantization in units of $4e^2/h$ has also been observed in Si inversion layers by Wang *et al* [18]. However, split-gate devices made from Si/SiGe heterostructures should be better suited to study ballistic transport in Si than inversion layers because of the electrons' much larger elastic mean free path as well as the easier tunability of the width of the constriction.

The flatness of the plateaus and the exactness of the quantization is not as accurate as, for example, in [2, 3]. This can be accounted for as follows: first, the elastic mean free path in the $Si/Si_{0.7}Ge_{0.3}$ sample used is still about an order of magnitude smaller than in GaAs/AlGaAs structures with comparable electron densities while the size of the imposed constriction is similar; second, we have assumed a gate voltage-independent serial resistance of the 2D leads. The subtracted value has been fitted and is slightly higher than $R_{4t}(V_{gate} = +0.4\ \mathrm{V})$. However, the electron mobility in this sample is very sensitive to changes of n_s [15]. It is therefore not too unrealistic to expect changes of the diffusive resistance of the 2D leads and of the Maxwell spreading resistance with the gate voltage even beyond the depletion threshold. The relatively small total number of observable plateaus is caused by the nature of the band structure in this material system. With an effective mass $m^* = 0.19m_0$ one naturally expects smaller subband spacings as compared to GaAs structures. The Fermi energy $E_F = 2\pi\hbar^2 n_s/m^* g_s g_v = 2.80$ meV is also reduced by almost a factor of six for the same carrier density.

Figure 2 also shows the conductance at magnetic fields $B = 1.1$, 2.3 and 4.5 T as a function of the applied gate voltage. These fields correspond roughly to filling factors $\nu = 16$, 8 and 4 respectively in the 2D leads. Suitable serial resistances and the influence of the Hall effect on the measured four-terminal resistances were taken into account [19]. The most obvious feature, as indicated by the arrows and the plateau indices, is the monotonic shift of the plateaus to more positive gate voltages with increasing magnetic field. This is the expected magnetic depopulation of hybrid subbands [8, 9] and will be analysed in detail below. It is also clear from figure 2 that the plateaus become flatter and more pronounced as B is increased. Backscattering in the constriction is suppressed at higher fields since the electron would have to be scattered from an edge state on one side of the point contact to an edge state on the opposite side. Furthermore, the shoulder at the depletion threshold becomes less and less pronounced. This may be explained by the expected transition from local to global adiabaticity [20].

Most interesting is the appearance of additional plateaus in between steps with integer subband indices. Contrary to GaAs/AlGaAs structures, the spin and valley degeneracies of 2DEGs in Si/SiGe heterostructures are resolved successively at rather small magnetic fields, as has been previously shown [21]. Conductance steps of $2e^2/h$ appear when the spin degeneracy is resolved. There are already some hints of this effect at $B = 1.1$ T. The curve at $B = 2.3$ T also exhibits some structure involving a conductance quantization of e^2/h, indicating the lifting of the valley degeneracy. Both effects are fully observable at $B = 4.5$ T when the electrons in the 2D leads completely fill the final Landau level: there are steps at e^2/h ($V_{gate} \leqslant -1.3$ V), $2e^2/h$ ($V_{gate} \approx -0.9$ V), $3e^2/h$ ($V_{gate} \approx 0$) and $4e^2/h$ ($V_{gate} \geqslant +0.3$ V). In addition, oscillations between the $2e^2/h$ and $3e^2/h$ plateaus can be observed at $B = 2.3$ T and $B = 4.5$ T. These are reminiscent of Aharonov–Bohm oscillations reported upon in point contacts by van Loosdrecht *et al* [22] and Wharam *et al* [23] and will be discussed in more detail elsewhere.

We have simulated the magnetic depopulation of the 1D subbands [8] using a square well potential of width W with a potential barrier of height E_c in the constriction while the Fermi energy E_F remained fixed at its 2D value. Ignoring spin and valley splittings a semiclassical expression for the number N of occupied subbands at given B, W and E_c is [8]

$$N = \mathrm{int}[\![(\tilde{k}_F l_c/\pi)\{\arcsin(W/2l_c) + (W/2l_c)[1-(W/2l_c)^2]^{1/2}\}]\!] \qquad \text{if } W < 2l_c$$
$$N = \mathrm{int}(\tilde{k}_F l_c/2 + 1/2) \qquad \text{if } W > 2l_c. \qquad (1)$$

Here $l_c = \hbar\tilde{k}_F/eB$ is the cyclotron radius and $\tilde{k}_F = \sqrt{2m^*(E_F - E_c)}/\hbar$ a modified Fermi wavevector reflecting the appearance of a potential step at the entrance to the QPC. An effective 2D carrier density in the point contact can then be defined as $n_c = g_s g_v \tilde{k}_F^2/4\pi$, which varies linearly with E_c. As pointed out in [8], equation (1) may have a discontinuity at $W = 2l_c$ and has an accuracy of ± 1.

Figure 3 displays the number of occupied 1D subbands versus the inverse of the applied magnetic fields for four different gate voltages. The symbols represent actual conduction data points while the full lines were calculated from equation (1) using the parameters listed in table 1. The potential barrier E_c was determined first from the high-field data when the quantization is almost entirely magnetic in nature. The width W was then derived from the value of G at $B = 0$ (symbols on the right-hand y scale). Good qualitative and reasonable quantitative agreement is found between the data and our simple model in spite of the difficulties involved in the exact determination of G at low magnetic fields. These, once again, arise from the relatively small number of observable plateaus as described above. We would like to stress that our model does not include any effects related to the lifting of the spin or valley degeneracies. Nevertheless, some of the data have been

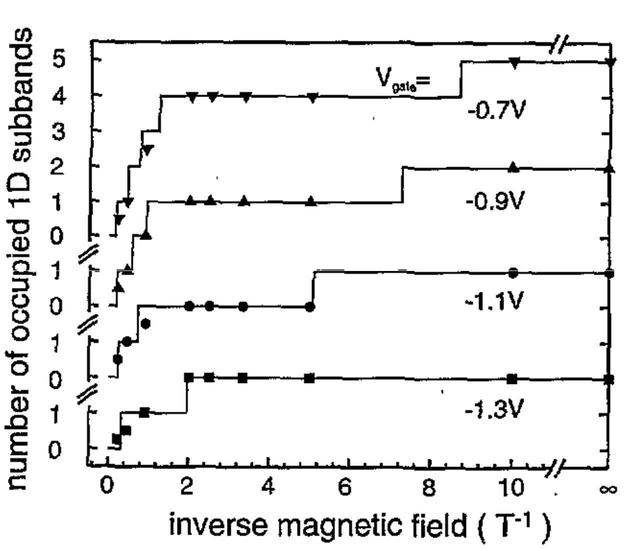

Figure 3. Number of occupied 1D subbands versus the inverse of the applied magnetic fields for gate voltages $V_{gate} = -0.7$, -0.9, -1.1 and -1.3 V. The full lines were calculated from equation (1) using the parameters listed in table 1. Spin and valley splittings were not taken into account. The symbols represent conduction data points. The lines are offset vertically for clarity.

Table 1. Barrier height E_c and well width W used to calculate the full lines in figure 3 from equation (1) for different gate voltages. n_c is an effective 2D carrier density in the constriction.

V_{gate} (V)	E_c (meV)	W (nm)	n_c (10^{11} cm^{-2})
−0.7	0.78	157	3.19
−0.9	1.17	140	2.59
−1.1	1.51	118	2.05
−1.3	1.83	95	1.54

plotted at non-integer occupation numbers whenever such liftings were observable in the experiments.

A more realistic potential shape in the constriction than our simple model would of course be a parabola with a flat bottom, as has been used by Wharam *et al* to model their data in [9]. As the effective width of the QPC is reduced the potential becomes more and more parabola-like and its bottom is finally raised in energy near pinch-off. Laux *et al* have obtained similar results in self-consistent calculations of a split-gate device [24]. Our model neglects the shape of the potential walls. Nevertheless, we believe that the error which is thereby introduced leads only to large quantitative disagreement at gate voltages close to 'pinch-off' [10]. It is thus interesting to consider the values of E_c and W as a function of the gate voltage. One empirically finds an almost linear dependence. At the threshold voltage $V_{gate} = -0.1$ V the barrier height E_c is about zero and at 'pinch-off' a little less than the Fermi energy. Also, the extrapolated well width W at zero gate bias is about 230 nm. A similar ratio of the geometric and effective widths in gated wires fabricated from the same wafer was recently reported by Holzmann *et al* [15] based on backscattering data. We therefore conclude that our model describes the potential in the QPC adequately well as long as we are not too close to 'pinch-off'.

In conclusion, we have studied ballistic transport through a QPC in a $Si/Si_{0.7}Ge_{0.3}$ heterostructure. The conductance G is found to be quantized in units of $i \times 4e^2/h$ in the absence of a magnetic field with i being the number of occupied subbands. By applying a perpendicular field, magnetic depopulation of hybrid magnetoelectric subbands is observed which can be simulated using a square well potential with a barrier in the constriction. In sufficiently high perpendicular magnetic fields additional conductance steps of $2e^2/h$ and e^2/h appear when the spin and valley degeneracies of the material system are lifted.

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